ELEMENTARY MECHANICS

FOR THE

PRACTICAL ENGINEER

ENGINEERS' STUDY COURSE FROM POWER

BY

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FIRST EDITION

McGRAW-HILL BOOK COMPANY, INC.

239 WEST 39TH STREET, NEW YORK

6 BOUVERIE STREET, LONDON, E. C.

1915

THE MAPLE PRESS YORK PA

PREFACE

The subject matter of this book is an exact reproduction of a series of thirty lessons which appeared in *Power* as a part of the Engineers' Study Course. These lessons were intended as "Home Study" for those who had followed the previous courses given in *Power*. Each lesson was so arranged that the average "home study" student could thoroughly master the material given each week, before proceeding with the next lesson. To drive home the principles given, five study questions were inserted each week and the student was urged to solve these problems before referring to the answers which appeared in the next lesson. The fact that these questions combined with their answers formed a vital part of the original course is the author's only excuse for the manner in which they appear in the text.

The aim of the entire course was to present only those principles of mechanics which could be directly applied to the various phases of power plant operation; and the problems were selected with this same point in mind. These questions were intended to arouse in the reader a desire to know the "*why*" and the "*wherefore*" of every machine or piece of apparatus with which he might come in contact during his daily work.

THE AUTHOR.

BROOKLYN, N. Y.
May, 1915.

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FOR THE

PRACTICAL ENGINEER

CHAPTER I

ELEMENTARY MECHANICS

The purpose of this section of the course, as of all that have preceded it, is to give a working knowledge of the fundamentals of mechanics sufficient for all practical needs. For this reason, only an elementary discussion of mechanics will be attempted.

To begin with, a few definitions will be given to fix the meaning of such terms as will be used from the start, then will follow simple explanations of the principles involved and practical applications of these principles will be given as examples or exercises, which will be made as pertinent as possible to power-plant problems.

To the average person the application of a principle is more interesting than a mere understanding of the principle by itself. For this reason the first few lessons, which will deal with definitions and statements of laws, may seem rather dry, but the student is urged to follow closely these introductory lessons so that he may more readily grasp the applications that will come later. Many will find that much of the early part of the course is familiar to them, but to others there will be new terms or at least new light on their meaning.

DEFINITIONS

Mechanics is that branch of science which has to do with forces and their action on bodies tending to produce a state of motion, change of motion, or condition of rest of these

bodies. At the outset it is essential that the student clearly understand that he is concerned only with forces and that all bodies will be considered as rigid, irrespective of the forces acting upon them.

The study of forces is divided into two general subjects, *kinetics* which deals with the forces that produce or change the motion of bodies and *statics* which deals with forces that keep a body at rest or in a state of equilibrium. The term *dynamics* is commonly used to include both kinetics and statics.

Force is that which produces, or tends to produce, motion or change of motion of bodies. It manifests itself to the feelings by a tension or pull, and by a compression or push. In other cases it manifests itself by an attraction or repulsion. Assume a man pulling on a rope—it is evident that the man can exert no pull on the rope unless the rope be attached at the other end to a resistance. An engine remains at rest on its foundation only because the foundation exerts an upward pressure equal to the weight of the engine. Stated in the familiar law of Newton, "*to every action there is an opposite and equal reaction*." Hence no single force can be exerted without there resulting an equal and opposite reaction. Another example is that of the pressure of the steam in a steam boiler. Suppose the steam pressure in the boiler is 100 lb. per sq. in. This means that on every square inch of surface in the boiler there is a pressure of 100 lb. The reaction is furnished by the material of the boiler, which, by virtue of its strength for each square inch is exerting a force of 100 lb. in resistance to rupture.

Gravity is the force by which all bodies are drawn to the earth. The force with which the earth attracts any body is called the *weight* of that body. Weight must not be confused with *mass* which will be discussed in a later lesson.

The unit of force is the weight of one pound or simply the pound.

Forces are represented graphically by straight lines; the *direction* of the force by an arrowhead placed at the end of the

line and by the angle which the force makes with a given reference line; the *magnitude* of the force by the length of the line. Three things are then necessary to completely determine a force:

(1) The point of application of the force.
(2) The direction of the force.
(3) The magnitude of the force.

In Fig. 1, O represents the point of application of the force P. The angle a gives the direction of the force with respect to a horizontal reference line OX, and the arrow-head at B indicates that the force is acting from O to B and not from B to O. The length of the line OB indicates the magnitude of the force P. Thus if 1 in. of length represents 50 lb. and the line OB is 2 in. long, then the force is

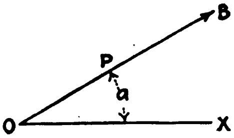

Fig. 1.

$$P = 2 \times 50 = 100 \text{ lb.}$$

COMPONENTS OF FORCES

The effect of a force in any given direction is called the *component* of the force. For convenience in dealing with problems, forces are generally resolved into their horizontal and vertical components. The component of a force can never be greater than the force itself and no force can have a component at 90 deg. to the line of action of the force. It must be evident, then, that a force exerts its maximum effort in its own line of action. When an engine is on dead center the pressure of the steam on the piston has no component tending to rotate the crankpin for the simple reason that at the given instant the motion of the crankpin is at right angles to the motion of the piston.

A force may have an infinite number of components. These components may be determined either graphically or by the aid of equations.

In Fig. 2, let OX and OY represent the horizontal and vertical reference lines drawn through the point of application O of the force P. Let the length of the line OS represent the magnitude of the force P. The components of the force P are found graphically as follows: From the point S drop a perpendicular SC onto the line OX forming the intercept OC. The length of this intercept OC is the horizontal component of the force P. In like manner the length of the line OD represents the vertical component of the force P. It would be possible to find other components of P, but for the present only horizontal and vertical components will be considered.

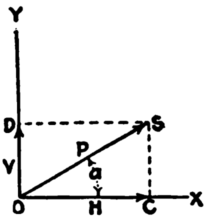

FIG. 2.

Let

H = Horizontal component OC of the force P.
V = Vertical component OD of the force P.
a = Angle which the force P makes with the horizontal.

Then by construction $OSCO$ is a right-angled triangle and from the laws of trigonometry the values of H and V can be determined in terms of the force P and the angle a, or

$$H = P \cos a \tag{1}$$

and

$$V = P \sin a \tag{2}$$

Problem 1.—A force of 200 lb. makes an angle of 30 deg. with the horizontal. Find the horizontal and vertical components of the force.[1]

Solution.—In Fig. 3, let OS represent the given force P = 200 lb. to scale; let angle SOX = 30 deg.

[1] NOTE.—In the solution of problems in mechanics the student will find the work very much simplified if he will make a neat sketch showing the relative position and magnitude of all the forces involved, as well as their direction.

Construct the rectangle *ONSMO* with *OS* as one diagonal. Then the line *OM* will represent the horizontal component *H*, and the line *ON* the vertical component. These lines can be measured and will give the numerical value of the components, or equations (1) and (2) can be applied and the components found. Thus, $H = P \cos a = 200 \times \cos 30$ deg. $= 200 \times 0.866 =$ 173 lb., and $V = P \sin a = 200 \times \sin 30$ deg. $= 200 \times 0.5 = 100$ lb.

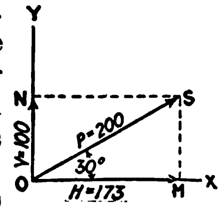

FIG. 3.

If the components of a given force are known the magnitude and direction of the force can be determined as follows: In Fig. 2, since *OSCO* is a right-angled traingle,

$$P^2 = \overline{SC}^2 + \overline{OC}^2$$

(the square of the hypotenuse equals the sum of the squares of the other two sides), but $SC = OD = V$ and $OC = H$; therefore

$$P^2 = H^2 + V^2$$

or

$$P = \sqrt{H^2 + V^2} \tag{3}$$

The tangent of the angle is

$$\tan a = \frac{SC}{OC} = \frac{V}{H} \tag{4}$$

From the last equation the angle itself may be found with the aid of a table of natural trigonometric functions.

Problem 2.—The horizontal component of a force is 30 lb. and the vertical component is 40 lb. Find the direction and magnitude of the force.

Solution.—Lay off $OD = 40$ lb. and $OC = 30$ lb. to scale, as in Fig. 4.

Draw *DS* parallel to *OC* and *CS* parallel to *OD* and draw the line *OS* which will represent the unknown force *P*. Then

$$P^2 = OD^2 + OC^2 = 40^2 + 30^2 = 1600 + 900 = 2500$$

Therefore,

$$P = \sqrt{2500} = 50 \text{ lb.}$$

$$\tan a = \frac{SC}{OC} = \frac{40}{30} = 1.333+$$

and the angle whose tangent is 1.333 is found to be 53 deg.

To aid in the problems that will follow later in the course, all components acting either to the right or upward will be

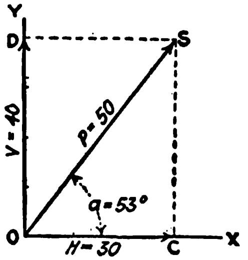

FIG. 4.

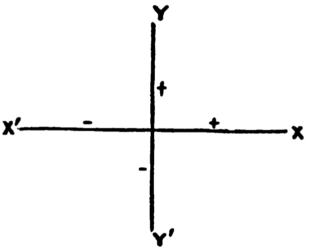

FIG. 5.

considered as positive forces (+), and all forces acting downward or to the left will be considered as negative (−), as indicated in Fig. 5.

Forces which pass through a common point are called "*concurrent forces*," and those whose lines of action do not pass through a common point are called "*nonconcurrent forces*."

Study Questions

1. What is the broad term for the action of forces on bodies?
2. What is force; how does it manifest itself?
3. When steam is admitted to the cylinder of an engine, where does the pressure find its reaction?
4. What is the component of a force?
5. What is meant by the terms gravity and weight?

Answers

1. Mechanics.
2. Force is that which produces, or tends to produce, motion or change of motion of bodies; it manifests itself by tension or compression, and attraction or repulsion.

3. In the strength of the cylinder walls and head, and by the resistance offered by the piston.

4. The component of a force is the effect of that force in any given direction.

5. Gravity is the force by which all bodies are attracted to the earth; the weight of a body is the measure of the force exerted by gravity on that body.

CHAPTER II

RESULTANTS OF FORCE

Any single force which produces the same effect on a body as the combined action of two or more forces is called the *resultant* of those forces. Take, for example, two men pulling on a vertical hoist. Assume each man exerts a pull of 50 lb. Their combined effort is 100 lb. A single pull of 100 lb. would then have the same effect as the combined pulls of the two men. In all cases where the forces are parallel and in the same direction the resultant is easily found by adding together the single forces. When the forces make an angle with each other the process of finding the resultant force is not so simple.

The resultant of a system of forces can be found either *graphically* or *algebraically*. The meaning of resultant force may be made very obvious by an easily performed experiment. Secure two small pulleys, a few pieces of stout cord and several small weights, and arrange the apparatus as shown in Fig. 6. Suppose, for example, the weight at the point A is 5 lb., at point B 3 lb., and at the point C 6 lb. Place a heavy piece of cardboard back of the cords and draw a line parallel to the line CF and make it proportional to the weight of 5 lb., according to some assumed scale; in like manner draw another line parallel to the cord CB and make it proportional to 3 lb. to the same scale. Construct the parallelogram $CFDEC$ by drawing DE parallel to CF and DF parallel to CB. Measure the line CD and, if the experiment has been carefully performed, it will be found that the line CD will measure 6 lb. to the given scale, and it will further be found that the line CD is vertical. Therefore if the weights W_1 and W_3 were replaced by a single vertical force acting upward at the point C and equal to 6 lb., this force would keep

the weight of 6 lb. from falling. It is to be noted that the resultant force is acting upward. The force of 6 lb. acting downward is called the *equilibriant* or the force producing a state of rest.

By varying the weights and changing the lengths of the cords (so as to secure different angles) the student can find the resultant for many other combinations, and he will always

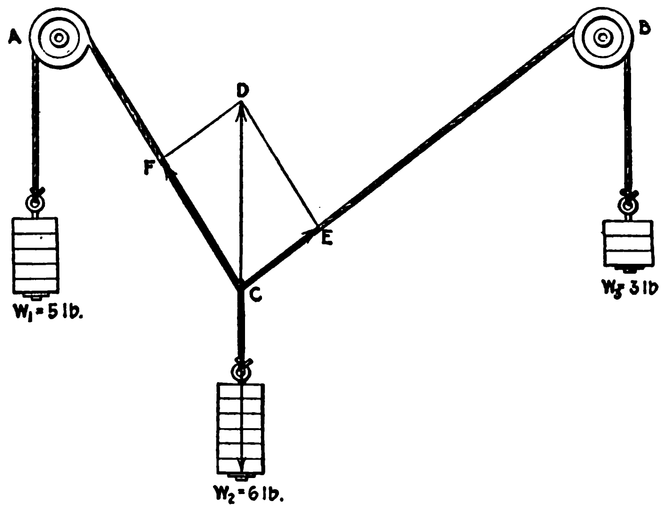

FIG. 6.

find it to be represented by the diagonal of the parallelogram formed by the two forces. From these simple experiments the following law can be stated:

If two forces acting at a point on a given body be represented in magnitude and direction by the adjacent sides of a parallelogram, the resultant of these two forces will be represented in magnitude and direction by the diagonal of the parallelogram passing through this point.

Another truth demonstrated by the above experiment is that the resultant force lies in the same plane with the forces to which it is equivalent. To find the resultant force by a

graphical method requires great care in the drawing of all lines and angles and even then a certain degree of error is apt to creep in. For this reason many prefer finding the resultant force by a formula.

In Fig. 7 assume two forces P_1 and P_2 acting at the point O and making an angle of a degrees with each other. Construct the parallelogram $OADBO$ and, from what has pre-

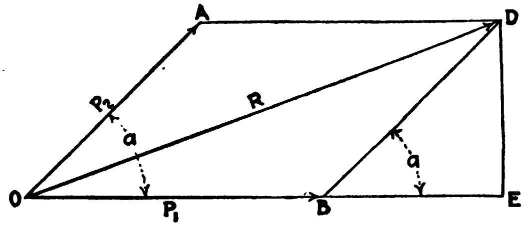

FIG. 7.

ceded, it must be evident that the diagonal OD is the desired resultant. What will be the value of R in terms of P_1, P_2 and the angle a? From the point D drop the perpendicular DE to the line OB (extended to E). There will then be formed the right-angled triangle $ODEO$, and from the law of squares, the square of the hypotenuse OD is equal to the sum of the squares of OE and ED. But $BD = OA = P_2$ and $DE = BD \sin a = P_2 \sin a$. Also $BE = BD \cos a = P_2 \cos a$

Then $OE = OB + BE = P_1 + P_2 \cos a$.

Therefore, since $\overline{OD}^2 = \overline{OE}^2 + \overline{DE}^2$,

$$\begin{aligned} R^2 &= (P_1 + P_2 \cos a)^2 + (P_2 \sin a)^2 \\ &= P_1^2 + 2P_1P_2 \cos a + P_2^2 \cos^2 a + P_2^2 \sin^2 a \\ &= P_1^2 + 2P_1P_2 \cos a + P_2^2 \qquad (5) \end{aligned}$$

since $(\cos^2 a + \sin^2 a) = 1$ (see Trigonometry).

Assume various values for the angle a and see how the values of R change. For example, if $a = 90$ deg. then $\cos a = 0$ and $R^2 = P_1^2 + P_2^2$ (equation 1, Chapter I) If $a = 45$ deg. then $\cos a = \frac{1}{2}\sqrt{2}$ and $R^2 = P_1^2 + P_2^2 + \sqrt{2}\, P_1P_2$.

The following table should be verified by the student.

If $a = 0$ deg. then $R^2 = P_1^2 + P_2^2 + 2\,P_1P_2$ or $R = P_1 + P_2$
$a = 30$ deg. then $R^2 = P_1^2 + P_2^2 + \sqrt{3}\,P_1P_2$
$a = 60$ deg. then $R^2 = P_1^2 + P_2^2 + P_1P_2^2$
$a = 120$ deg. then $R^2 = P_1^2 + P^2{}_2 - P_1P_2$
$a = 135$ deg. then $R^2 = P_1^2 + P_2^2 - \sqrt{2}\,P_1P_2$
$a = 150$ deg. then $R^2 = P_1^2 + P_2^2 - \sqrt{3}\,P_1P_2$
$a = 180$ deg. then $R = P_1 - P_2$

By a careful study of the parallelogram law it will be noted that the resultant always lies nearest the greater force. If the angle between the forces is less than 90 deg. the resultant is greater than either single force; if the angle is greater than 90 deg. the resultant can only be greater than one of the forces.

A third and perhaps simpler method of determining the resultant of two forces (and this method can be applied where there are more than two forces) is to resolve the forces into their horizontal and vertical components, and then find the algebraic sum of all the horizontal components and the algebraic sum of all the vertical components. Thus all the forces are replaced by a single vertical force and a single horizontal force and the resultant can be found from equation (1), given in Chapter I.

Problem.—Two forces of 80 lb. and 120 lb. make an angle of 30 deg. with each other. Determine the resultant by each of the three methods discussed.

(1) **Graphical Solution.**—Draw the lines *OB* and *OA* at an angle of 30 deg. with each other (see Fig. 8). Lay off the force $P_1 = 120$ lb. along the line *OA* to an assumed scale. (It is advisable to always draw one of the forces either horizontal or vertical to simplify the drawing.) Similarly lay off $P_2 = 80$ lb. along the line *OB* to the same scale as used for P_1. Construct the parallelogram and draw the diagonal R. The length of this line multiplied by the scale used will give the desired resultant force. By careful measurement of Fig. 8 the resultant will be found to equal 193 lb.

(2) **Solution by the General Equation.**—Referring to the table, when $a = 30$ deg., $R^2 = P_1^2 + P_2^2 + \sqrt{3}\,P_1P_2$. In this problem $P_1 = 120$ and $P_2 = 80$. Substituting these values in the equation there results,

$$R^2 = 120^2 + 80^2 + \sqrt{3} \times 120 \times 80$$
$$= 14{,}400 + 6400 + 1.732 \times 9600 = 37{,}427$$
$$R = \sqrt{37{,}427} = 193.4$$

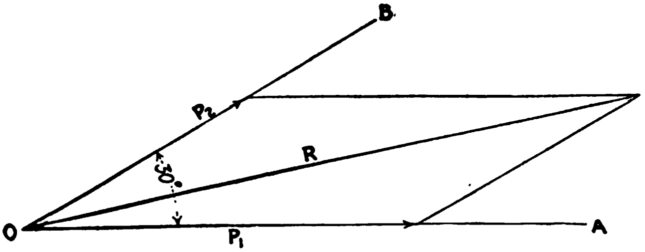

FIG. 8.

This value is a trifle larger than that found by the graphical solution. If a larger scale had been used in the graphical solution the error would have been reduced and the two values would have probably checked.

(3) **Solution by Components.**—The horizontal component of the force P_1 is 120 lb. and its vertical component is 0. The horizontal component of the force P_2 is

$$80 \times \cos 30 \text{ deg.} = 80 \times 0.866 = 69.28 \text{ lb.}$$

and the vertical component is

$$80 \times \sin 30 \text{ deg.} = 80 \times 0.5 = 40 \text{ lb.}$$

Hence the sum of the vertical components is $0 + 40 = 40$ lb. and the sum of the horizontal components is $120 + 69.28 = 189.28$ lb. From the table when two forces make an angle of 90 deg. with each other $R^2 = P_1^2 + P_2^2$. Substituting the above values there results,

$$R^2 = 40^2 + 189.28^2 = 1600 + 35{,}827 = 37{,}427$$
Therefore, $R = \sqrt{37{,}427} = 193.4$.

This value must necessarily check with the value found by the general equation. By using two of the above methods the student can always check his results and thus be certain of their accuracy.

When the resultant of a series of forces is desired it may be found graphically by a continuous application of the parallelo-

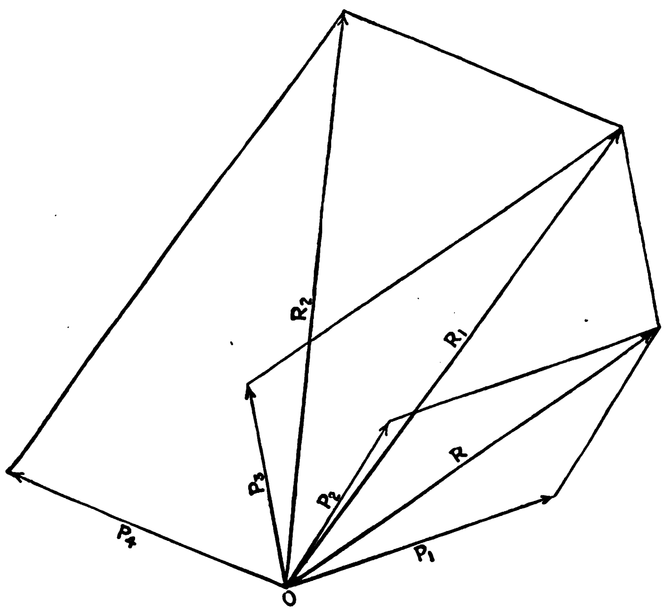

FIG. 9.

gram law. Thus in Fig. 9 let the forces P_1, P_2, P_3 and P_4 be acting on a body at the point O. Construct a parallelogram on P_1 and P_2, giving the resultant R. On R and P_3 construct another parallelogram giving the resultant R_1. Construct a parallelogram on R_1 and P_4 giving as the final resulant the force R_2. This final resultant would be the same irrespective of the order in which the forces are taken, the truth of which the student can readily prove by working the problem.

Study Questions

6. Define the term resultant force and state three methods by which the resultant may be found.

7. A connecting-rod makes an angle of 12 deg. with the horizontal. If the total pressure on the piston at this point is 6000 lb., what is the connecting-rod thrust?

8. In the above problem determine the pressure between the crosshead and the guides.

9. In problem 8 find the force tending to rotate the crankpin and also the force tending to break the crank. (The force tending to rotate the crankpin must act at 90 deg. to the line of the crank.) Assume that the crank makes an angle of 90 deg. with the horizontal.

10. Four forces act on a given body at the point *O*. The first force is horizontal and equals 50 lb. The second force makes an angle of 60 deg. with the horizontal and equals 60 lb. The third force makes an angle of 30 deg. with the horizontal and equals 40 lb. The fourth force makes 120 deg. with the horizontal and equals 45 lb. Find the magnitude and direction of the resultant.

Answers

6. The resultant of two or more forces is a single force that will produce the same effect on a body as the combined effect of the other forces. The resultant can be found first, by a graphical solution; second, by the use of the general formula for the resultant of two forces; and, third, by resolving the forces into their horizontal and vertical components and using the resultant components as two forces acting at 90 deg. to one another.

7. To solve this problem the student must bear in mind that when three forces act on a body any one force may be considered as the resultant of the other two forces. The pressure exerted by the steam is transmitted by the piston to the piston rod and thence to the connecting-rod, through the crosshead. The connecting-rod exerts a reaction on the crosshead, which produces a pressure between the crosshead and the guides, and this pressure is at right angles to the direction of motion of the crosshead. The thrust in the connecting-rod can then be considered as the resultant of the reaction exerted by the guide on the crosshead, and the pressure of the steam on the piston rod. In Fig. 10, let the length of the line *OA* represent the load *P* on the piston equal to 6000 lb. Let *OB* represent the line of the connecting-rod and *ON* the direction of the pressure between the crosshead and the guide. The direction of the pressure between the crosshead and the guide is given by *ON*, but *OM* represents the reaction offered by the guide.

Construct the parallelogram $OABMO$. The diagonal OB will give the desired thrust in the connecting-rod, and its value can be found from the right-angled triangle $OABO$.

$$\text{As } \cos 12 \text{ deg.} = \frac{OA}{OB}$$

$$OB = \frac{OA}{\cos 12 \text{ deg}} = \frac{6000}{0.978} = 6140 \text{ lb.}$$

This problem demonstrates the fact that when the lines of action of three forces are known and only the value of one force is given, the values of the other two can readily be found by applying the parallelogram law.

8. The pressure between the crosshead and the guides is represented by the line ON which by construction is equal to AB, which in turn is the altitude of the triangle $OABO$. In the triangle $OABO$, tan 12 deg. $= \dfrac{AB}{OA}$.

Therefore, $\quad AB = OA \tan 12 \text{ deg.} = 1276 \text{ lb.}$

9. When a force acts on a body its point of application can be assumed anywhere in the line of action of the force. Consider the thrust in the connecting-rod transferred from the point O to the point C, and let its value be represented by the length of the line CF (Fig. 10). As explained

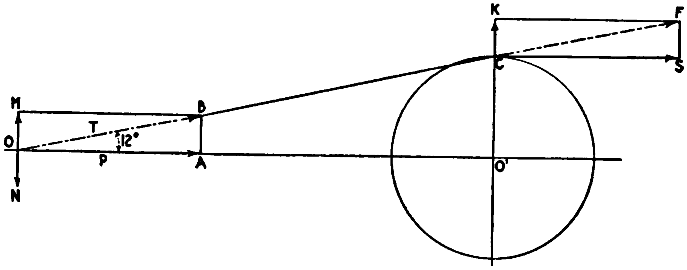

FIG. 10.

in Chapter I, any force can be resolved into components along any desired lines. Hence, resolve the force CF into two components, one at right angles to the line of the crank $O'C$, and the other parallel to the crank. To do this draw the line CS at right angles to $O'C$ and extend the line of the crank to K. Draw the parallelogram $CKFSC$. The length of the line CS will then represent the force tending to rotate the crankpin, and CK will represent the force which, at this instant, produces a tension in the crank.

By construction the angle FCS must equal 12 deg., since CF is parallel to OB, and CS is parallel to OO'. Also $CF = OB = 6140$ lb.

$$CS = CF \cos 12 \text{ deg.} = 6000 \text{ lb.}$$

and

$$FS = CF \sin 12 \text{ deg.} = 1276 \text{ lb.}$$

10. First make a neat drawing of all the forces acting, representing their magnitudes by the length of the lines (to a given scale) and showing their direction by laying off the necessary angles. Thus in Fig. 11 draw the line $OA = 50$ lb.; the line OB (making an angle of 30 deg. with the

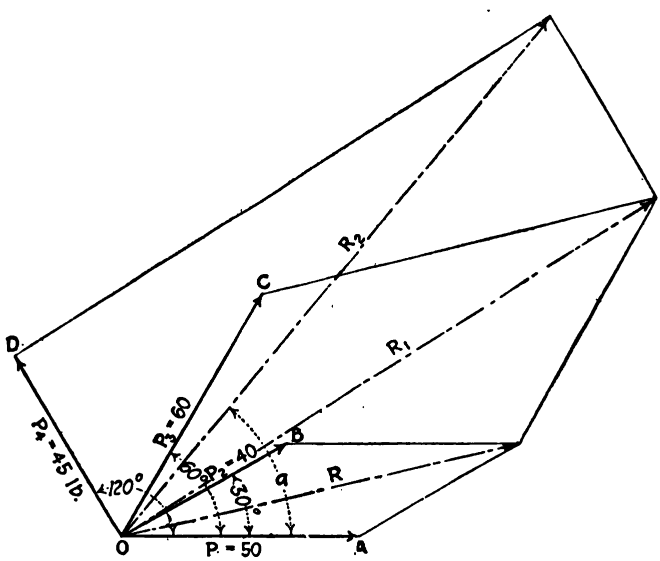

FIG. 11.

horizontal) = 40 lb.; the line OC (making an angle of 60 deg. with the horizontal) = 60 lb.; and the line OD (making an angle of 120 deg. with the horizontal) = 45 lb. On the forces P_1 and P_2 construct the parallelogram of forces giving the resultant R; in like manner combine R and P_3, giving the resultant R_1; and finally combine R_1 and P_4, giving the resultant R_2. By careful measurement R_2 is found to equal 143.5 lb.

The resultant might be found by resolving all the forces into their horizontal and vertical components.

Thus the sum of the horizontal components is

$$H = 50 + 40 \cos 30 \text{ deg.} + 60 \cos 60 \text{ deg.} + 45 \cos 120 \text{ deg.}$$
$$= 50 + (40 \times 0.866) + (60 \times 0.5) = (45 \times 0.5) = 92.14.$$

The sum of the vertical components is

$$V = 0 + 40 \sin 30 \text{ deg.} + 60 \sin 60 \text{ deg.} + 45 \sin 120 \text{ deg.}$$
$$= 0 + (40 \times 0.5) + (60 \times 0.866) + (45 \times 0.866) = 110.9.$$

but $$R^2 = H^2 + V^2$$

Therefore $$R^2 = 92.14^2 + 110.9^2 = 20.795.$$
$$R = 144.2 \text{ lb.}$$

The value of R is a trifle higher than that found from the graphical solution, but is more accurate.

The direction of the resultant force R_2 is found from the equation,

$$\tan a = \frac{V}{H} = \frac{110.9}{92.14} = 1.204$$
$$a = 50 \text{ deg.} - 20 \text{ min.}$$

Therefore, the resultant force equals 144 lb. and makes an angle of 50 deg., 20 min. with the horizontal.

FREE BODY

In the problem just solved the student has doubtless noted that no reference was made to the size or shape of the various parts of the steam engine, but only the forces exerted by the various parts were considered. When all the parts of a machine, structure, or body are replaced by the corresponding forces, or reactions, exerted by the parts, the whole is spoken of as a *free body*. In the problem on the engine the wristpin at the point O receives the thrust of the connecting-rod and also the thrust of the piston rod, yet for the solution of the problem it is easier to neglect the source of the force, or the parts transmitting the force, and consider the forces only. Take, for example, a beam resting on two columns, and carrying several concentrated loads. From the standpoint of mechanics the beam, the form of the loads, and the nature of the columns can be neglected, so that the problem would take the form shown in Fig. 12. The reaction exerted by one

column has been replaced by a single upward force R_1 at the point B; the concentrated load at D, which might be the weight of a machine or another column, has been replaced by a single force W_1 acting downward; the load C has been replaced by

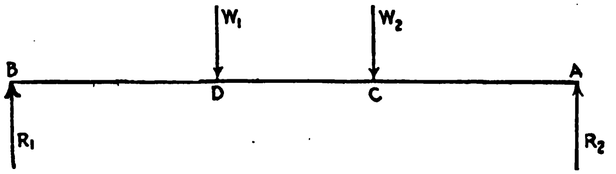

FIG. 12.

the force W_2 and the reaction of the column A by the single force R_2 acting upward. In the solution of any problem dealing with forces, the student will find his work much simplified, if he will first analyze his problem, then isolate all forces and reactions and solve as if dealing with a free body.

CHAPTER III

EQUILIBRIUM

When any number of forces act on a body so as to produce a state of rest or uniform motion of a body the forces are said to be in *equilibrium* or balanced. An oil can is resting on a shelf. Think of the can as a free body and what is the result? The weight of the can will be replaced by a single force acting downward, while the resistance offered by the shelf, due to its strength, will be replaced by a single force acting upward. These two forces must have the same point of application and be equal to each other but acting in opposite directions. The least number of forces that can produce equilibrium is, therefore, two. Hence *when two forces act on a body to produce a state of equilibrium it follows that these forces must have the same point of application and be equal in magnitude and opposite in direction.* So long as a body remains at rest it is self-evident that the forces are balanced. Suppose, however, a body is in motion—are the forces in equilibrium as before? Take, for instance, a steam engine running at a uniform rate of speed. The pressure of the steam on the piston is just sufficient to balance the load on the engine, and to take care of all friction in the bearings, pins, etc. As long as these forces are balanced the engine continues to rotate at a uniform rate of speed, and will continue to do so unless some of the forces are changed. For example, the least drop in the steam pressure will cause the forces to become unbalanced and the engine will start to slow down; or if the pressure increases the equilibrium is again destroyed and the engine will speed up. Hence the condition of equilibrium of a body, whether at rest or in a state of uniform motion, can only be changed by the addition of an extra force.

What then are some of the conditions of equilibrium for concurrent forces acting in the same plane? Suppose all the forces acting on the body are resolved into their horizontal and vertical components. It must be evident that if there is to be equilibrium of a body the *resultant of all the forces acting on the body must equal zero,* or, put in the form of an equation,

$$R = 0 \tag{6}$$

For the resultant to be zero the algebraic sum of all the horizontal components must equal zero, and the algebraic sum of all the vertical components must equal zero. Therefore,

$$R^2 = \Sigma H^2 + \Sigma V^2 = 0 \tag{7}$$

hence

$$\Sigma H = 0 \tag{8}$$

$$\Sigma V = 0 \tag{9}$$

Σ stands for "summation of," or "algebraic sum of."

Study Questions

11. What is meant by the term "free body"?
12. Are the forces acting on a body at rest in equilibrium?
13. Can the forces acting on a body in motion be in equilibrium?
14. State the general conditions for equilibrium for concurrent forces.
15. Make a sketch showing the crankshaft of a vertical side-crank engine as a free body. Assume the engine on the top dead center.

Answers

11. When a body is regarded as acted upon by forces only it is called a "free body."
12. Yes.
13. Yes.
14. The conditions of equilibrium are that (1) the resultant = 0; (2) the summation of the horizontal components = 0; and (3) the summation of the vertical components = 0.
15. In Fig. 13 the force P_1 represents the force exerted by the steam on the piston, transmitted through the piston and connecting-rod, to the crankpin. The force P_2 might represent either the weight of a flywheel, or the rotating field, or armature, of a generator. The forces R_1 and R_2 represent the reactions offered by the main bearings, and the

force P_3 could be either the weight of a pulley, or the weight of a flywheel, depending upon the type of engine and the nature of the load.

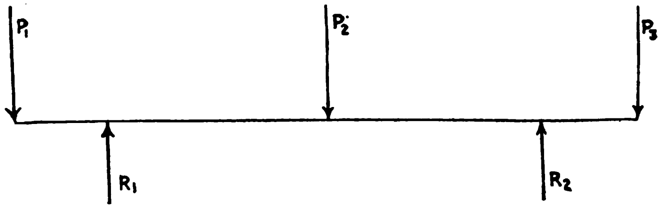

FIG. 13.

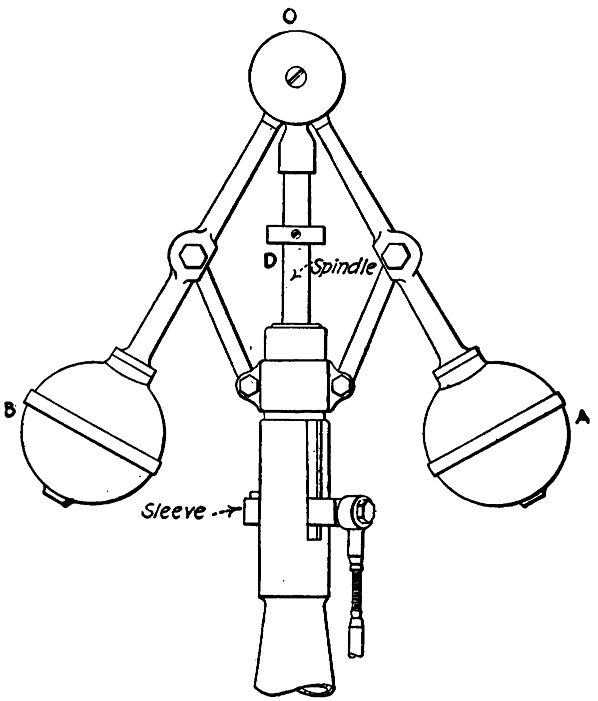

FIG. 14.

If the three conditions of equilibrium, as stated at the close of last week's lesson, are thoroughly mastered by the

student, but little trouble will be experienced in dealing with forces which produce a state of uniform motion or rest of a body. Referring to Fig. 10 in Chapter II, the force T is the resultant of OM and the force P, but this resultant T is balanced by the corresponding reaction of the crankpin on the connecting-rod. Therefore, since the forces are all in balance the resultant must equal 0. To find the force, then, required to balance a system of forces it is necessary (1) to isolate all

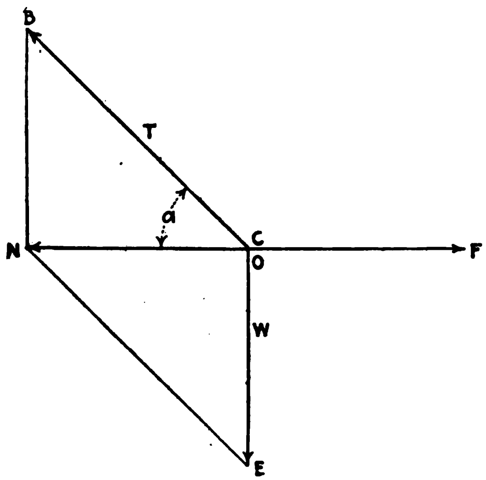

FIG. 15.

the forces from the body on which they are acting, (2) to resolve these forces into their horizontal and vertical components, (3) from the components thus found to determine the resultant force, and (4) then the desired force will be a force which is equal and opposite to the above resultant.

To demonstrate this principle take the ordinary type of flyball governor, as shown in Fig. 14. As the engine speeds up, or slows down, the balls B and A move farther away from, or closer to the spindle OD. When the engine is running at a uniform speed the governor balls remain at a fixed distance from the spindle.

Let Fig. 15 represent one of the balls as a free body. The line *OB* is drawn parallel to the line *OA* and represents the resistance offered by the arm. The line *OE* is vertical and proportional to the weight of the ball *A*, and the horizontal line *OF* indicates the centrifugal force tending to pull the ball *A* in a straight line. Construct the parallelogram *BOENB* and draw the diagonal *ON*, which is the resultant of the forces *T* and *W*. The centrifugal force exerted by the ball *A* must then be equal and opposite to the resultant *ON*, and when such a condition holds true all the forces are in balance and will continue to be so until the speed of the engine changes. This will change the value of the centrifugal force *C* and then the arm *OA* will take a new position until the forces again come into a state of equilibrium.

TRIANGLE OF FORCES

When three forces acting in the same plane keep a body in equilibrium the resultant of any two of the forces must be equal and opposite to the third force. For this to be true the third force must pass through the same point as the other two. *Therefore, when three forces acting in the same plane produce a state of equilibrium, these forces must pass through a common point.*

Example.—Let Fig. 16 represent a common form of wall crane and assume the load as acting at the point *A*. This load *W* causes (1) a tension or pull in the tie-rod *AC* which is counteracted by the rod exerting a corresponding reaction *T*, from *A* to *C*; and (2) a compression or push in the beam *AB*, which is counteracted by the reaction *P* of the beam from *B* to *A*. From the previous discussion it follows that the resistance offered by *BA* is equal and opposite to the resultant of the tension in the tie-rod *AC* and the weight *W*.

Let Fig. 17 represent the point *A* as a free body. The line *OG* replaces the resistance offered by the beam *BA*, the line *W* replaces the weight *W*, and the line *ON* replaces the resistance

offered by the rod AC. From the point N draw the line NH equal and parallel to the line W. Draw HO equal and parallel

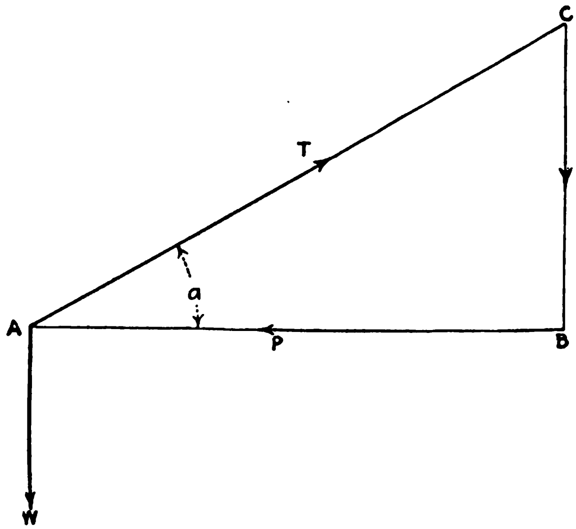

Fig. 16.

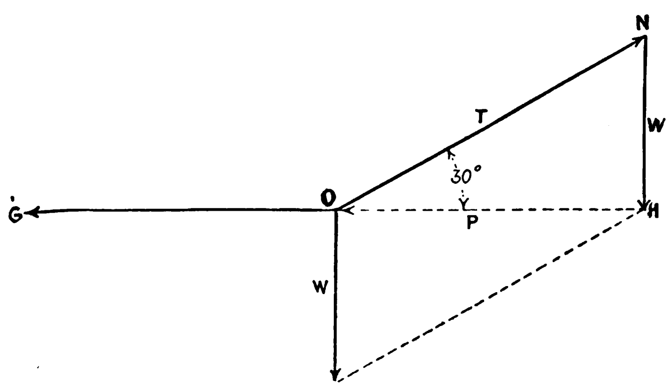

Fig. 17.

to OG. There is thus formed the triangle $ONHO$ in which it will be noted that the force T is acting from O to N, the force W from N to H, and the force P from H to O. In other

words, the three forces acting on the point O have been replaced by a triangle which is called a *force triangle.* Hence the following rule can be stated:

When three forces acting at a given point on a body are in equilibrium, these forces can be represented in direction and magnitude by the sides of a triangle taken in order, if the sides of the triangle are parallel to the respective forces. Conversely, if three forces acting on a body can be represented in direction and magnitude by the sides of a triangle taken in order, the forces will be in equilibrium.

This law is simply another application of the parallelogram of forces. The important part of the law is that the sides must be taken in the same order, or in the same way around. This law does away with the necessity of resolving forces into their horizontal and vertical components. Thus in the triangle $ONHO$ (Fig. 17) the angle NOH must equal the angle a (which is known) and the side NH equal the known weight or load W. The hypotenuse ON will then equal the unknown tension T in the tie-rod AC and the base OH will give the compression in the beam AB.

Example.—Suppose the angle a (Fig. 16) is 30 deg. and the load $W = 1000$ lb.; what is the tension T in the rod AC, and what is the compression P in the beam AB? Use the force triangle of Fig. 17, letting the angle $NOH = 30$ deg. and the line $NH = 1000$ lb.

$$\sin a = \frac{NH}{ON} = \frac{W}{T}$$

therefore,

$$T = \frac{W}{\sin a} = \frac{1000}{0.5} = 2000 \text{ lb.}$$

Also

$$\tan a = \frac{NH}{OH} = \frac{W}{P}$$

therefore,

$$P = \frac{W}{\tan a} = \frac{1000}{0.577} = 1732 \text{ lb.}$$

This problem is comparatively simple because the triangle contains a right angle.

Study Questions

16. Two forces of 80 and 100 lb. act on a body and make an angle of 60 deg. with each other. Find the direction and magnitude of a third force to produce equilibrium. Assume the force of 100 lb. as horizontal.

17. In Fig. 16 the tension T must not exceed 2000 lb. If the angle a is 35 deg., what is the maximum load that can be suspended from the point A?

18. In Fig. 15, if the weight $W = 40$ lb. and the centrifugal force $C = 30$ lb., determine the tension in the arm OB.

19. In Fig. 16, let the weight W be suspended at a point midway between the points A and B. Will the tension in the tie-rod AC be increased or decreased?

20. Will the pressure at the point B remain horizontal? If not, how could its line of action be determined?

Answers

16. In Fig. 18, let OA equal the force of 100 lb. and OB the force of 80 lb. Construct the parallelogram $OACBO$ and draw the diagonal OC to give the resultant force R.

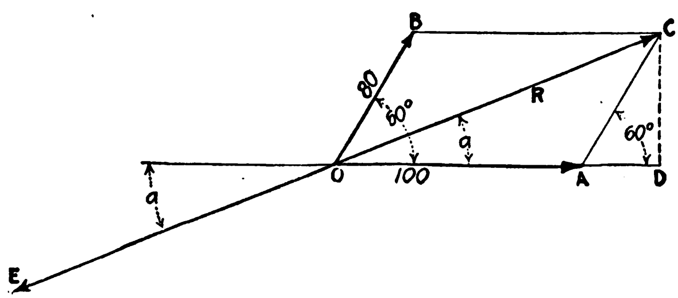

FIG. 18.

From equation 5, Chapter II,

$$R^2 = 100^2 + 80^2 + 2 \times 100 \times 80 \times \cos 60 \text{ deg.};$$
$$= 10{,}000 + 6400 + 8000 = 24{,}400;$$
$$R = 156.2 \text{ lb.}$$

Draw CD perpendicular to OD then $CD = AC \sin 60$ deg. $= 69.3$ and $AD = AC \cos 60$ deg. $= 40$

$$\tan a = \frac{CD}{OD} = \frac{69.3}{140} = 0.495$$

therefore, a = 26 deg. 20 min. The equilibriant of a system of forces is equal and opposite to the resultant of these forces. Therefore, the line OE, which is equal and opposite to OC, is the desired force and makes an angle of 26 deg. 20 min. with the horizontal.

17. Refer to Fig. 17. In this case the angle NOH = 35 deg. The line $ON = T$ = 2000 lb. and the line NH represents the unknown load W.

$$\frac{NH}{NO} = \sin 35 \text{ deg.} = 0.574$$

hence

$$NH = NO \sin 35 \text{ deg.} = 2000 \times 0.574 = 1148 \text{ lb.}$$

18. The tension in the link OB is the resultant of the weight W of the ball, and the centrifugal force C. In the triangle $BONB$ (see Fig. 15), the force OB is equal and opposite to the resultant of the weight W and the centrifugal force OF.

Also

$$NB = OE = W = 40 \text{ and } ON = OF = 30$$

therefore

$$OB^2 = T^2 = ON^2 + NB^2 = 30^2 + 40^2 = 2500$$

or

$$T = 50 \text{ lb.}$$

19. The tension in the tie-rod AC will be decreased since part of the weight W is now taken by the point B, where previously it was all taken by the point A.

20. The pressure at B will no longer be horizontal since part of the weight W is acting at the point B. The pressure on the point B is equal to the resultant of the pressure in the strut AB and the part of the weight taken by the point B. Draw a vertical line through the center of the line AB until it intersects the line AC. A line connecting this point with B will give the direction of the pressure on the point B. Proof: If three forces acting on a body produce equilibrium, the lines of action of these forces pass through a common point.

CHAPTER IV

LAW OF SINES

From the lessons in trigonometry the student doubtless recalls the rule that in *any triangle any side is proportional to the sine of the angle opposite the given side.* (*Law of sines.*) This law can be used to solve the force triangle when it contains no right angle. For example, take the case of a weight W suspended from two tie-rods which are pivoted at the points A and B and make angles with the horizontal, as shown in Fig. 19. What will be the tensions in the rods OA and OB? Draw the triangle $OACO$ having the side AC parallel to OB and the side OC proportional and parallel to the weight W. In this triangle the side OA is proportional to the tension T_2 and the direction of the force is from O to A; AC is proportional to T_1 and its direction is from A to C; the side CO is proportional to the weight W and its direction is from C to O. Hence the three forces acting on the point O are represented by the sides of the triangle $OACO$ taken in order. By construction the angle CAB equals 30 deg. since AC is parallel to OB. Therefore, the angle ACO = 60 deg. (Note: The sum of all the angles of any triangle = 180 deg.) For the same reason the angle COA = 90 deg.—45 deg. = 45 deg. The angle opposite the side CO, or W, is 75 deg.; opposite the side AO, or T_2, the angle is 60 deg.; and opposite AC, or T_1, the angle is 45 deg. Now apply the law of sines and there result these equations

$$\frac{T_1}{\sin 45 \text{ deg.}} = \frac{W}{\sin 75 \text{ deg.}}$$

also

$$\frac{T_2}{\sin 60 \text{ deg.}} = \frac{W}{\sin 75 \text{ deg.}}$$

hence

$$T_1 = \frac{W \sin 45 \text{ deg.}}{\sin 75 \text{ deg.}} = \frac{1000 \times 0.707}{0.966} = 732 \text{ lb.}$$

and

$$T_2 = \frac{W \sin 60 \text{ deg.}}{\sin 75 \text{ deg.}} = \frac{1000 \times 0.866}{0.966} = 897 \text{ lb.}$$

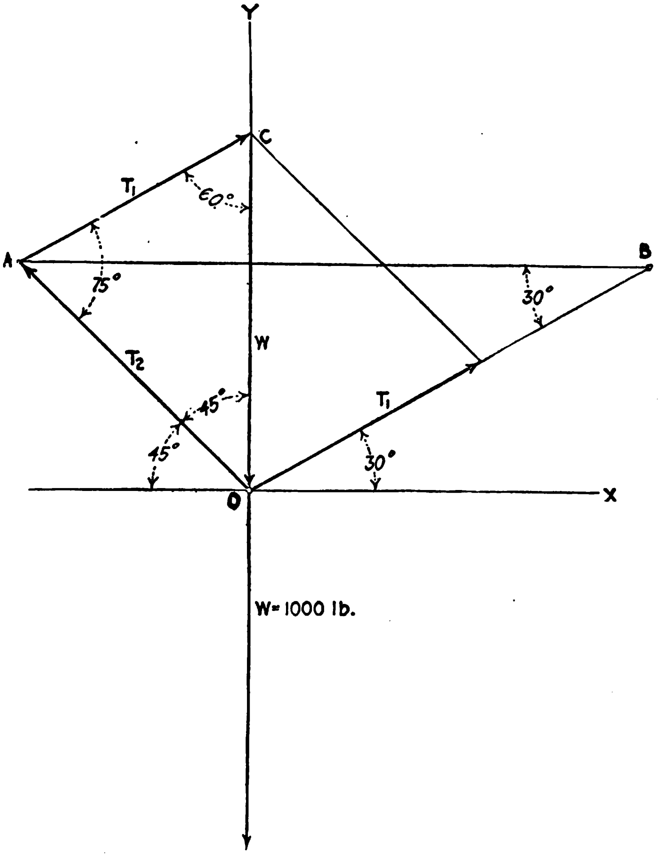

FIG. 19.

For those who are familiar with the law of sines the above solution will be easy. For the benefit of those not familiar

with the law the problem will now be solved by resolving the forces into their horizontal and vertical components and applying the laws $\Sigma H = O$ and $\Sigma V = O$. Use the point O (Fig. 19) as the origin and draw the two axes OX and OY through this point. The horizontal component of the tension $T_1 = T_1$ cos 30 deg. and the vertical component $= T_1$ sin 30 deg. Likewise the horizontal component of $T_2 = - T_2$ cos 45 deg. (this is minus because the component is acting to the left). The vertical component $= T_2$ sin 45 deg. The horizontal component of $W = W$ cos 90 deg. $= 0$ and the vertical component $= - W$ (minus because W is acting downward).

Therefore,

$$\Sigma H = T_1 \cos 30 \text{ deg.} - T_2 \cos 45 \text{ deg.} + 0 = 0$$

then

$$0.866\ T_1 - 0.707\ T_2 = 0$$

hence

$$T_1 = \frac{0.707\ T_2}{0.866} = 0.8164\ T_2$$

also

$$\Sigma V = T_1 \sin 30 \text{ deg.} + T_2 \cos 45 \text{ deg.} - W = 0$$

then

$$0.5\ T_1 + 0.707\ T_2 - 1000 = 0$$

or

$$0.5\ T_1 + 0.707\ T_2 = 1000$$

Substitute in this equation the value of $T_1 = 0.8164\ T_2$ and there results

$$0.5 \times 0.8164\ T_2 + 0.707\ T_2 = 1000$$

or

$$1.1152\ T_2 = 1000$$
$$T_2 = 897 \text{ lb.}$$

and

$$T_1 = 0.8164\ T_2 = 0.8164 \times 897 = 732 \text{ lb.}$$

By a comparison of the two solutions it is seen that the method of sines is much shorter than the method of com-

ponents and for this reason the student is urged to study the law of sines and to combine it with the triangle of forces in the solution of subsequent examples.

POLYGON OF FORCES

Thus far only two- and three-force problems have been considered, but frequently problems will arise in which there are four or more forces acting at a given point. In such cases the principle of the triangle of forces is extended to include the *polygon of forces.*

Rule.—*If any number of forces in the same plane, acting at a given point on a body produce a state of equilibrium, these forces can be represented in magnitude and direction by the sides of a polygon taken in order. Conversely, if any number of forces*

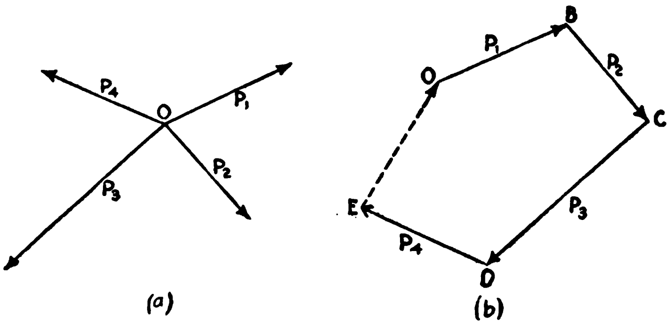

FIG. 20.

in the same plane acting at a given point on a body can be represented by the sides of a polygon taken in order, the forces will be in equilibrium.

The polygon of forces can be used to determine the force necessary to balance a system of forces. In Fig. 20a, the four forces P_1, P_2, P_3 and P_4 act at a given point O. Do these forces produce equilibrium? In Fig. 20b, draw OB parallel and equal to P_1, BC parallel and equal to P_2, CD parallel and equal to P_3 and DE parallel and equal to P_4. There is thus formed the figure $OBCDE$ which is not a closed polygon and

hence the forces are not in equilibrium. A line drawn from the point E to the point O will represent the magnitude and the direction of the force required to balance the system. The student can prove the truth of this statement by applying the parallelogram law to determine the resultant of the forces P_1, P_2, P_3 and P_4 as explained in Chapter II. The force necessary to produce equilibrium will be equal and opposite to the resultant thus found, and must be equal to the line EO.

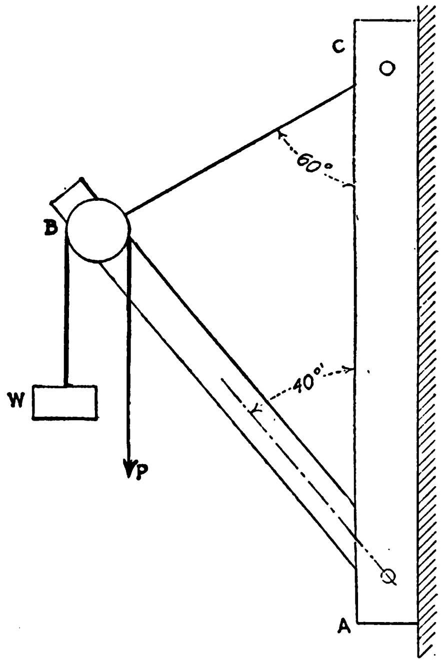

FIG. 21.

Study Questions

21. In Fig. 21 determine the tension in the tie-rod BC when the weight W = 600 lb. (NOTE: P = 600 lb.) Solve by the aid of a force triangle.

22. Find the compression in the strut AB.

23. What produce the reactions necessary for equilibrium in Fig. 21?

24. Using the magnitude and directions of the forces given in problem 10, and, replacing the resultant force of 144 lb. by an equal and opposite force, construct a force polygon.

25. Are the forces in problem 24 in equilibrium? Give a reason for your answer.

Answers

21. Consider the point B as a free body. In Fig. 22 let the line AC represent the total vertical load $= (P + W) = 1200$ lb. The line BC

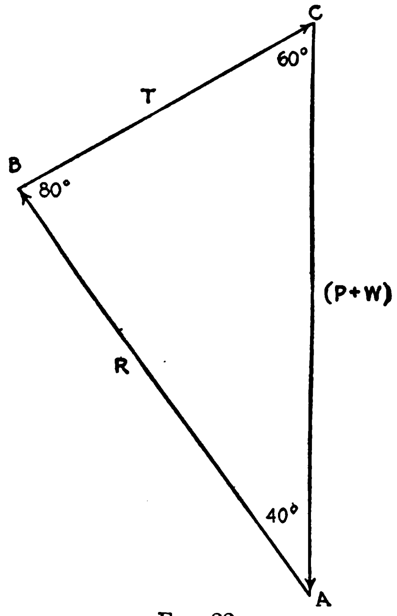

FIG. 22.

drawn parallel to the rod BC will represent the tension T in the rod, and AB drawn parallel to the strut AB will represent the compression R in the strut. The angle CAB being 40 deg. and the angle ACB 60 deg., the angle CBA must equal $(180 - 60 - 40 =)$ 80 deg.

From the law of sines

$$\frac{T}{\sin 40 \text{ deg.}} = \frac{(P + W)}{\sin 80 \text{ deg.}}$$

or

$$T = \frac{(P + W) \sin 40 \text{ deg.}}{\sin 80 \text{ deg.}} = \frac{1200 \times 0.643}{0.985} = 783 \text{ lb.}$$

22. Again applying the law of sines there results the equation,

$$\frac{R}{\sin 60 \text{ deg.}} = \frac{(P + W)}{\sin 80 \text{ deg.}}$$

or

$$R = \frac{(P + W) \sin 60 \text{ deg.}}{\sin 80 \text{ deg.}} = \frac{1200 \times 0.866}{0.985} = 1055 \text{ lb.}$$

23. The reaction counteracting the tension T is the strength of the rod which is dependent upon the size and material of the rod. Like-

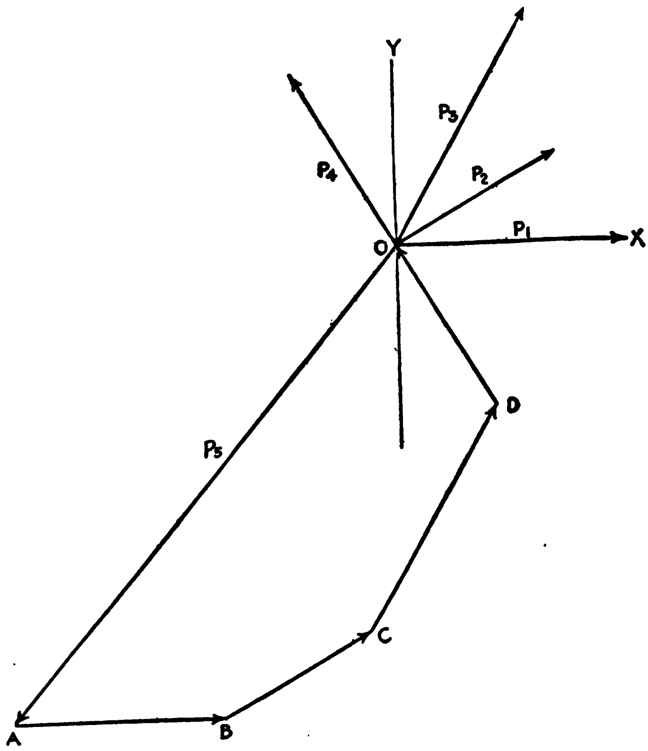

FIG. 23.

wise the strength of the strut AB produces the necessary reaction to counteract the thrust R.

24. In Fig. 23 draw the forces P_1, P_2, P_3 and P_4 with the directions and magnitudes given in problem 10. Draw the force P_5 equal and

opposite to the resultant which was found to be 144 lb. Starting with the point O and using the force P_5 as a base line draw a force polygon, making OA equal and parallel to P_5; AB equal and parallel to P_1; BC equal and parallel to P_2; CD equal and parallel to P_3; and the line OD equal and parallel to P_4.

25. All the forces must be in equilibrium since they can be represented by the sides of the polygon $OABCDO$ taken in order. If the line OD does not come out equal and parallel to the force P_4 there has been an error somewhere in the student's work.

CHAPTER V

PARALLEL FORCES

Parallel forces are those which act at different points of a body and have their lines of action parallel. If the forces act in the same direction they are called *like forces;* and if they act in opposite directions they are called *unlike forces.* The resultant of two like forces is equal to the *sum* of the forces and *acts in the same direction* as the forces; the resultant of two unlike forces is equal to the *difference* of the two forces and *acts in the direction of the greater force.* In the case of like forces the resultant will lie between the two

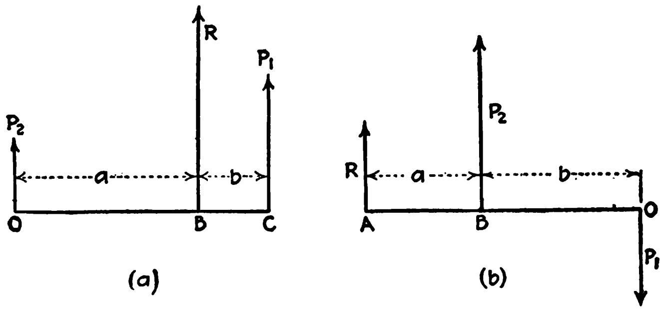

FIG. 24.

forces and in the case of unlike forces the resultant will lie outside of both forces. The above laws are almost axiomatic, or self-evident, and no proof will be given at present.

Parallel forces are simply an extension of the parallelogram law in which case the point of meeting of the forces is at an infinite distance from the body on which the forces are acting. In Fig. 24a the resultant R of the two like forces P_1 and P_2

equals $(P_1 + P_2)$ and acts at a point B such that the distances a and b are inversely proportional to the forces P_1 and P_2, or

$$\frac{P_1}{P_2} = \frac{a}{b}$$

In Fig. 24b the resultant R of the two unlike forces P_1 and P_2 is equal to $(P_2 - P_1)$ and acts at the point A such that

$$\frac{P_1}{R} = \frac{a}{b}$$

MOMENT OF A FORCE

Concurrent forces acting on a body tend to produce motion in the direction of the resultant force. Such motion is spoken of as one of translation, since the motion is in a straight line. Another kind of motion is that of rotation in which case the given force tends to rotate the body about a fixed axis. Thus the tension in a belt causes the pulley, on which it is acting, to rotate about its fixed center. Hence:

The tendency of a force to produce rotation of a body about a definite or imaginary axis is called the moment of that force.

The *measure* of the *moment* of a force is the *product of the force times the perpendicular distance from the given axis to the line of action of the force.* Generally the distances will be expressed in feet, the forces in pounds, and hence the moments will be given in foot-pounds.[1]

For convenience in the solution of problems, moments tending to produce rotation in the direction of the hands of a clock will be taken as positive (+) and all moments tending to produce counter-clockwise rotation as negative (−).[2]

[1] Many writers on mechanics prefer to use the term pound-feet for moments, to distinguish from the unit of work which is the foot-pound.

[2] Here again writers are not uniform, as some prefer to use clockwise rotation as negative. In all problems in this course clock-wise rotation will be taken as positive.

COUPLES

Two equal and opposite parallel forces acting at different points of a body produce what is known as a *couple.* The perpendicular distance between the forces is called the *arm* of the couple. Since the forces are equal and opposite in direction the resultant force is zero and therefore the action of a couple is not to produce translation, but to produce, or tend to produce, rotation of a body. The moment of a couple is equal to the product of one of the forces times the perpendicular distance between the forces, or times the arm of the couple. No single force can balance a couple and hence the equilibriant of a couple is another couple equal and opposite to the given couple.

The moment of a couple remains constant, irrespective of the point about which the moments of the forces are taken. This may be proved as follows:

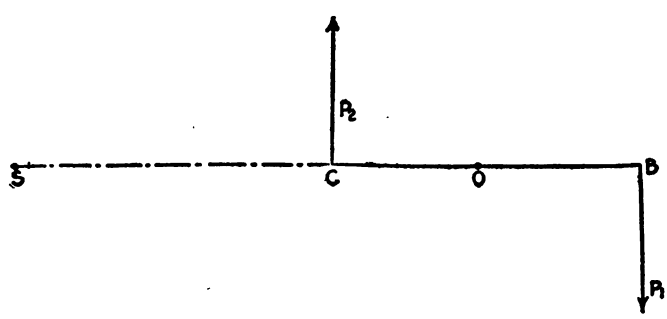

Fig. 25.

In Fig. 25, let the equal and opposite forces P_1 and P_2 produce a couple whose moment arm is CB. The moment of this couple about the point O equals

$$P_1 \times OB + P_2 \times OC$$
$$= P_1 (OB + OC) \text{ (since } P_1 = P_2)$$
$$= P_1 \times CB$$

Now assume some other point S. The moment of the couple about this point

$$= P_1 \times SB - P_2 \times SC$$
$$= P_1\,(SB - SC)\ (\text{since } P_1 = P_2)$$
$$= P_1 \times CB\ (\text{for } SB - SC = CB)$$

Therefore, the effect of a couple upon a rigid body is unaltered if it be transferred to any plane parallel to its own, the arm remaining parallel to its original position.

Study Questions

26. Two men carry a weight suspended from a horizontal pole. The one man exerts a pressure of 40 lb. and the other 60 lb. If the distance between the men is 10 ft., what is the magnitude and location of the weight? Neglect the weight of the pole.

27. A man turns the crank of a windlass. Is there a couple produced? If so, where is the second force?

28. The handle of a grindstone is 12 in. long. A man exerts a pressure of 30 lb. in the direction of motion of the handle. What is the turning moment produced?

29. A prony brake is clamped to the pulley of a motor. What is the turning moment or torque of the motor when a given armature current produces a net pull of 10 lb. at the end of the brake arm? The length of the brake arm is 24 in.

30. In question 9, Chapter II, determine the effective moment tending to rotate the crankshaft. Does this moment remain constant during a complete revolution of the engine? Assume the engine stroke to be 18 in.

Answers

26. In Fig. 26, let the line ABC represent the given pole and let the pressures exerted by the men be represented by the forces P_1 and P_2.

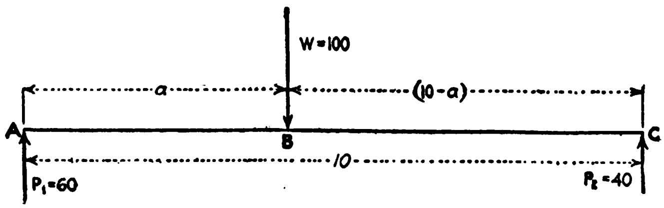

FIG. 26.

The resultant of these two forces is equal to 60 + 40 = 100 lb. Hence the weight W, which is equal and opposite to the resultant, must equal 100 lb. This weight is located nearest the man supporting the greater load. The distance a may be found from the equation,

$$\frac{a}{(10 - a)} = \frac{P_2}{P_1}$$

or

$$a = \frac{P_2}{P_1}(10 - a) = \frac{40}{60}(10 - a)$$
$$= \frac{400}{60} - \frac{40\,a}{60} = \frac{20}{3} - \frac{2\,a}{3}$$
$$3a = 20 - 2\,a \text{ or } 5\,a = 20$$

hence $a = 4$ ft. and $(10 - a) = 6$ ft.

27. This problem illustrates a special form of couple; namely, a couple in which one of the equal and opposite parallel forces is produced by the reaction of a fixed bearing. Thus the pressure exerted by the man on the crank is one force, the distance from the center of the handle to the center of the shaft is the arm of the couple, and the second force is the reaction offered by the bearing, without the existence of which, the man could exert no pressure at all on the crank.

28. The moment arm equals 12 in., the force is 30 lb. and, therefore, the turning moment (more commonly called *torque*) produced is equal to $12 \times 30 = 360$ in.-lb., or 30 ft.-lb.

29. In this problem the force is 10 lb. and the moment arm is 24 in.; therefore the turning moment is

$$24 \times 10 = 240 \text{ in.-lb., or } 20 \text{ ft.-lb.}$$

30. The effective force tending to produce rotation of the crankshaft was found in problem 9 to equal 6000 lb. The moment arm of the force is the length of the crank which is one-half the stroke of the engine or $\frac{1}{2} \times 18 = 9$ in.

Hence the turning moment is

$$9 \times 6000 = 54{,}000 \text{ in.-lb.}$$

This moment does not remain constant since the pressure of the steam on the piston varies and also the angle between the crank and the connecting-rod.

PRINCIPLE OF MOMENTS

In the discussion on concurrent forces it was shown that, for a state of equilibrium to exist, the sum of the horizontal and vertical components must equal zero. In the case of

nonconcurrent, or parallel, forces these conditions are not sufficient to insure equilibrium, for the sum of all the components might equal zero and yet there still might exist a tendency for the forces to produce rotation of the given body about a real or assumed axis. Hence the following additional condition must be fulfilled to produce equilibrium in a system of nonconcurrent forces:

When any number of forces acting on a body produce a state of equilibrium the algebraic sum of the moments of all these forces about any given point must equal zero. The converse of this rule does not necessarily hold true, for the sum of the moments might equal zero while there still existed either a resultant horizontal or vertical component through the point about which moments were taken.

It is to be noted that in the rule the term "any number of forces" was used without any statement being made as to the kind of forces—that is, whether concurrent or nonconcurrent. This means that the law applies equally as well to concurrent as to nonconcurrent forces, and in many instances problems can be solved more readily by moments than by the application of either the triangle or polygon of forces.

Thus in Fig. 16, Chapter III, suppose the perpendicular distance from the point B to the line AC was 18 in. and the distance AB was 30 in. Take moments about the point B. The force P acting in the strut AB *can have no moment about the point B since the force P passes through the point B* and as the moment arm is zero, the moment must also equal zero. The only forces, then, having moments about the point B are the tension T in the rod AC and the weight W acting at the point A. The moment of $T = 18 \times T$ and the moment of $W = 30 \times W$. From the condition of equilibrium previously stated the algebraic sum of these moments must equal zero, or $18\ T - 30\ W = 0$; hence

$$T = \frac{30\,W}{18} = \frac{5}{3}\,W$$

For all problems involving either concurrent or nonconcurrent forces then the three following equations must always be satisfied.

$$\Sigma H = 0$$
$$\Sigma V = 0$$

(see equations (8) and (9), Chapter III).

$$\Sigma \text{ moments} = 0 \qquad (10)$$

Since the resultant of a system of forces is a single force that will produce the same effect on a body as the combined action of the system it must be apparent *that the moment of the resultant is equal to the algebraic sum of the moments of all the other forces.*

A common application of this principle is in the determination of the reactions of the beam. In this case the pressure acting at either support or *pier* is the resultant of all the other forces. The beam remains at rest, due to the fact that the support exerts a reaction which is equal and opposite to the above resultant.

A practical application of this principle is made use of in the design of the main bearings of an engine. The load on each bearing can be found when the weight of the shaft and fly-wheel, the thrust of the connecting-rod, etc., are known, by taking moments about each bearing.

Problem.—A beam of 18-ft. span carries concentrated loads of 1000 lb., 2400 lb. and 600 lb. at distances of 4, 10 and 15 ft., respectively, from the left end of the beam. The beam is supported by a column placed at each end of the span. Find the load on each column, neglecting the weight of the beam.

In Fig. 27, let AE represent the beam of 18-ft. span and W_1, W_2 and W_3 the respective loads located as stated in the problem. Let R_1 and R_2 be the reactions exerted by the columns. The load acting on the column at E is the resultant of the reaction at the point A, and the loads W_1, W_2 and W_3.

The force counteracting this resultant is the reaction of the column at the point E indicated by the force R_2. For equilibrium the algebraic sum of the moments about any point

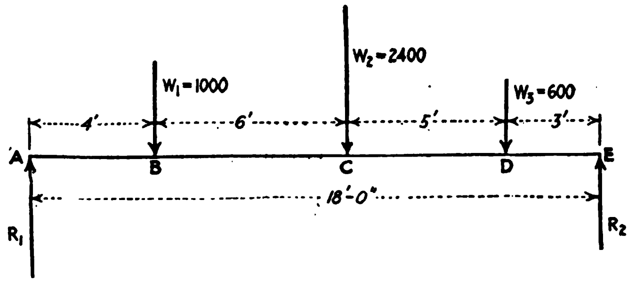

FIG. 27.

must equal zero. Taking moments about the point A there results the equation,

$$R_1 \times 0 + 4 \times 1000 + 10 \times 2400 + 15 \times 600 - 18 \times R_2 = 0$$

or

$$18 \times R_2 = 0 + 4000 + 24{,}000 + 9000 = 37{,}000$$

hence

$$R_2 = 2055 \text{ lb.}$$

In like manner taking moments about the point E there results the equation,

$$R_2 \times 0 - 3 \times 600 - 8 \times 2400 - 14 \times 1000 + 18 \times R_1 = 0$$

or

$$18 \times R_1 = 0 + 1800 + 19{,}200 + 14{,}000 = 35{,}000$$

hence $$R_1 = 1945 \text{ lb.}$$

The student should study these equations carefully so as to understand the use of positive and negative moments.

However, these equations do not fully satisfy the condition of equilibrium and the additional test of $\Sigma V = 0$ must be applied. From this law there results the equaton,

$$-1000 - 2400 - 600 + 2055 + 1945 = 0$$

or $$-4000 + 4000 = 0$$

Hence all the conditions of equilibrium are satisfied and the reaction of 2055 lb. at the point E and 1945 lb. at A will keep the beam at rest under the given loads. If any error had been made in the computation of R_1 and R_2 from the equations of moments it would have been discovered when the law of $\Sigma V = 0$ was applied.

After the student has acquired confidence in problems dealing with beam reactions he will find himself determining one reaction by taking moments, and then subtracting the reaction thus found from the algebraic sum of the vertical loads to find the remaining reaction. This is not to be commended as there will be no check on the work. If, however, moments are taken about each reaction and, if the sum of the reactions found from the equations of moments does not equal the sum of the vertical loads, it means that an error has been made and the necessary corrections may be made before proceeding farther in the design of the beam.

Study Questions

31. Fig. 28 is a diagrammatic sketch of the crankshaft of a vertical side-crank engine. The weight of the flywheel is 9000 lb. and acts at a point 65 in. from the right-hand bearing. The weight of the armature of the generator plus the magnetic pull is 15,000 lb. and acts at a point 41½ in. from the right-hand bearing. The thrust P on the crankpin is 45,000 lb. Assume the weight of the shaft as concentrated 53 in. from the right-hand bearing. Weight = 2000 lb.

Make a sketch showing the location of all the loads and the reactions.

32. Find the reactions R_1 and R_2 for the downstroke by taking moments about the points R_1 and R_2.

33. Find them for the upstroke in like manner.

34. Check the computations for 32 and 33 by making $\Sigma V = 0$.

35. At the instant shown in Fig. 28, does the pressure of the steam on the piston produce any turning moment on the crankshaft?

Answers

31. Fig. 29 shows the magnitude and location of all the vertical loads and reactions acting on the crankshaft, when the piston of the engine is on the top dead center. In this figure the forces are located both from the reaction R_1 and the reaction R_2. The purpose of this is to facilitate

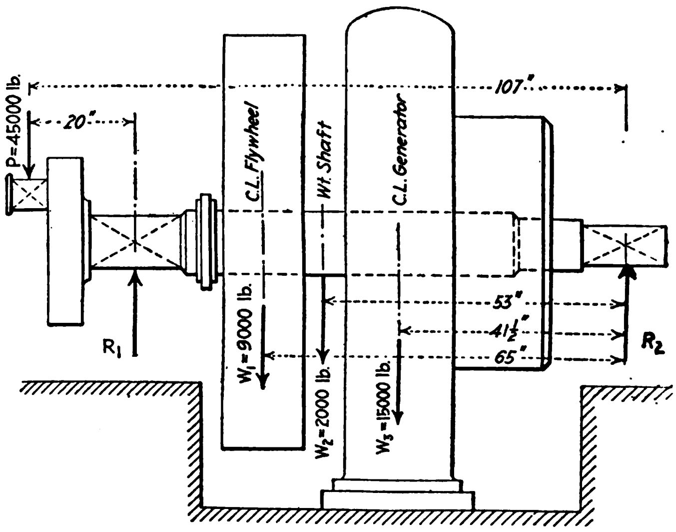

FIG. 28.

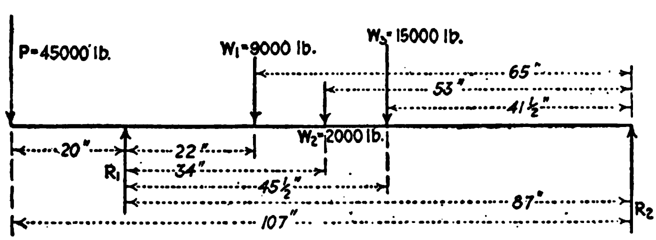

FIG. 29.

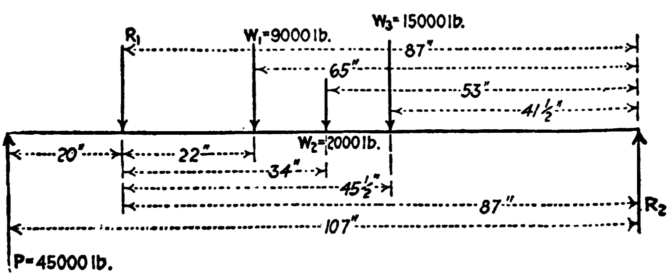

FIG. 30.

the taking of moments about either point, by clearly showing the moment arm of each force.

Fig. 30 shows the location of all the forces acting on the crankshaft when the piston is on the bottom dead center. In this case the direction of the force P is the reverse of what it was on the downstroke, and hence the reaction R_1 is downward instead of upward as before.

32. Take moments about the point R_1 (see Fig. 29).

Thus,

$$-87 \times R_2 + 45\tfrac{1}{2} \times 15{,}000 + 34 \times 2000 + 22 \times 9000 - 20 \times 45{,}000 = 0$$

or $$87 \times R_2 = 948{,}500 - 900{,}000 = 48{,}500$$

and $$R_2 = 558$$

To determine R_1, take moments about the point R_2; thus,

$$-41\tfrac{1}{2} \times 15{,}000 - 53 \times 2000 - 65 \times 9000 + 87 \times R_1 - 107 \times 45{,}000 = 0$$

or $$87 \times R_1 = 6{,}128{,}500$$

and $$R_1 = 70{,}442$$

33. Upstroke. Take moments about the point R_2 (Fig. 30).

Thus,

$$-41\tfrac{1}{2} \times 15{,}000 - 53 \times 2000 - 65 \times 9000 - 87 \times R_1 + 45{,}000 \times 107 = 0$$

or $$87 \times R_1 = 4{,}815{,}000 - 1{,}313{,}500 = 3{,}501{,}500$$

and $$R_1 = 40{,}247$$

To determine R_2, take moments about the point R_1; thus,

$$20 \times 45{,}000 + 22 + 9000 + 34 \times 2000 + 45\tfrac{1}{2} \times 15{,}000 - 87 \times R_2 = 0$$

or $$87 \times R_2 = 1{,}848{,}500$$

and $$R_2 = 21{,}247$$

34. The accuracy of the results found in problems 32 and 33 can be checked by applying the law $\Sigma V = 0$; thus,

$$\Sigma V = 45{,}000 + 9000 + 2000 + 15{,}000 - 70{,}442 - 558 = 0$$

or $$\Sigma V = 71{,}000 - 71{,}000 = 0$$

This proves that the values of R_1 and R_2 as determined in problem 32 are correct.

For the upstroke, problem 33, $\Sigma V = 0$; thus,

$$\Sigma V = 40{,}247 + 9000 + 2000 + 15{,}000 - 45{,}000 - 21{,}247 = 0$$

or

$$\Sigma V = 66{,}247 - 66{,}247 = 0$$

Hence the values of R_1 and R_2 are also correct.

35. The pressure of the steam on the piston, transmitted through the piston rod and the connecting-rod to the crank, does not produce any turning moment since there is no component of the connecting-rod thrust acting in the direction of motion of the pin.

CHAPTER VI

CENTER OF GRAVITY

A body may be considered as made up of a series of small particles. The weights of all these particles form a system of vertical parallel forces, and the resultant of this system must evidently equal the sum of these forces or the total weight of the body. The point at which this resultant force or weight acts is called the *center of gravity* of the body. As commonly defined *the center of gravity of a body is the point through which the line of action of the weight of the body always passes.*

In the following discussion the term center of gravity will be abbreviated to the symbol *c.g.* If a bar of uniform cross-section is suspended by a rope attached at one end of the bar it is evident that the bar will hang in a vertical, and not in a horizontal position; or if the rope is attached to the center, the bar will probably assume a horizontal position. Hence a vertical line drawn through the point of suspension of a body must always pass through the *c.g.* of the body.

The *c.g.* of bodies which are symmetrical with respect to a given point, and are of uniform density, will be at the given point. Thus the *c.g.* of a sphere is at its geometrical center and the *c.g.* of a circle is at its center. The student can readily think of many more examples. Therefore, in many cases the determination of the *c.g.* is simply a matter of inspection. In the case of unsymmetrical figures the *c.g.* may be found experimentally by making a template of cardboard to represent the body to a given scale. Next suspend the template, first from one point and then from another, and in each case draw a vertical line through the point of suspension. The intersection of these two vertical lines will locate the *c.g.* of the template, from which the *c.g.* of the body can readily be

found. Take, for example, the counterweight attached to the crank of certain forms of steam engines. Let the weight be of uniform thickness and its cross-section be of the form shown in Fig. 31, which is a template of the given weight. If the template be supported from the point O and the vertical line CD be drawn through the point of suspension, then the $c.g.$ of the body will lie somewhere on this line. Likewise, if

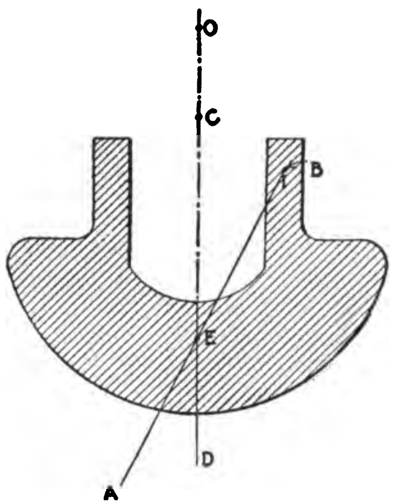

FIG. 31.

the template be suspended from the point B the $c.g.$ will lie on the line AB. Therefore, the $c.g.$ of the body will be found at the point E of intersection of the two lines CD and AB. This method is frequently used in the drafting room, as no computations are necessary.

If a body can be divided into regular geometrical figures, such as triangles, squares and rectangles, the $c.g.$ is best obtained by applying the law that the moment of the resultant (which is the total weight of the body) is equal to the sum of the moments of the other forces (the weights of each of the separate figures).

Problem.—At what point would a rope be attached to raise the piston and piston rod in Fig. 32 so that the rod will remain in a horizontal position?

Solution.—The rope must be attached at the center of gravity of the piston and rod considered as a single body. This point is found as follows: The weight of the piston rod

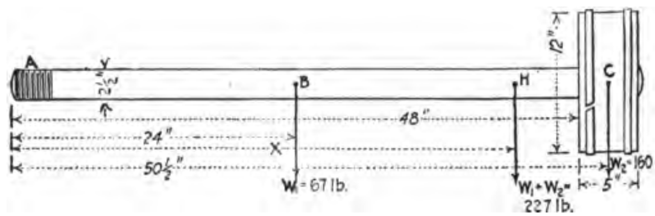

FIG. 32.

may be assumed as concentrated at its *c.g.*, which is evidently at the center of the rod, as at *B*. Likewise the weight of the piston is concentrated at its center *C* for the same reason. Let the unknown point be located at *H*, which is *x* in. from the point *A*, or end of the rod. Take moments about this point *A*.

Thus,

$$24 \times 67 + 50\tfrac{1}{2} \times 160 = 227 \times x$$

or

$$227\,x = 9688$$

and

$$x = 42.7 \text{ in.}$$

Problem.—Fig. 33 shows the cross-section of a riveter frame. Locate the *c.g.* of this section.

Solution.—This figure is symmetrical about its vertical axis, and hence its *c.g.* lies somewhere on the line *AB*, say, at the point *H*, *x* in. from the line *CD*. Divide the section into the three rectangles I, II and III. By inspection it is clear that the *c.g.* of each rectangle is at the point of intersection of the diagonals and also that the weight of each part is proportional to the area of the part. The moment of the

weight of each section about any given point will then be proportional to the moment of the area. In dealing with problems of this kind the moments of the areas are usually taken in order to determine the *c.g.*

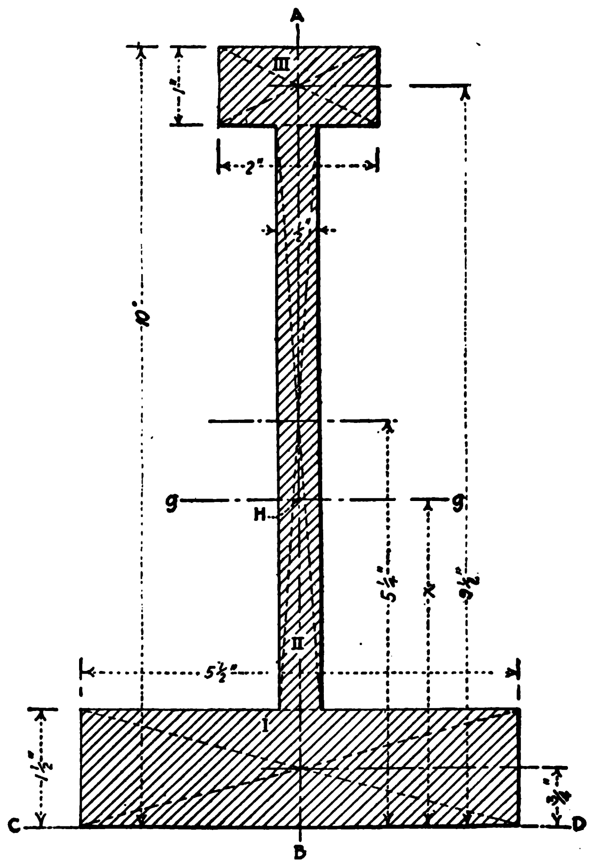

FIG. 33.

The *c.g.* of part III is located $9\frac{1}{2}$ in. from the line *CD*, of part II, $5\frac{1}{4}$ in. from *CD*, and of part I, $\frac{3}{4}$ in. from *CD*. The area of part I equals $5\frac{1}{2} \times 1\frac{1}{2} = 8\frac{1}{4}$ sq. in., the area of part

II equals $7\frac{1}{2} \times \frac{1}{2} = 3\frac{3}{4}$ sq. in., and the area of part III equals $2 \times 1 = 2$ sq. in.

Now take moments of the areas I, II and III about the line CD. The sum of these moments must equal the moment of the resultant (total area).

Thus,

$$\frac{3}{4} \times 8\frac{1}{4} + 3\frac{3}{4} \times 5\frac{1}{4} + 9\frac{1}{2} + 2 = 14 \times x$$

or
$$14x = 44.87$$

and
$$x = 3.2 \text{ in.}$$

Study Questions

36. Locate the center of gravity of a triangle.

37. Does the center of gravity of a body always lie within the body? Illustrate by an example.

38. A hole 4 in. in diameter is punched in a plate which is 15 in. square and of uniform thickness. The center of the hole is located 6 in. from either side. Find the *c.g.* of the punched plate.

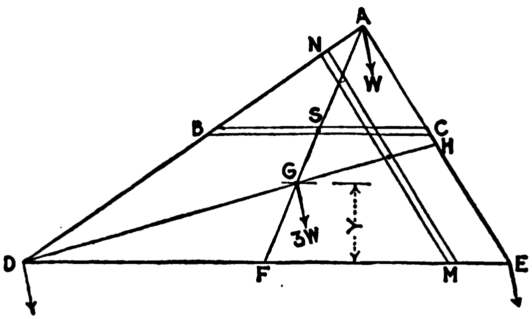

FIG. 34.

39. An iron ball 6 in. in diameter is attached to a rod 2 in. in diameter. If the length of the rod from the end to the point of contact with the ball is 18 in., find the *c.g.* of the ball and rod considered as a single body.

40. The frustum of a right cone is 4 in. high; the diameter of the top is 4 in.; the diameter of the base is 8 in. Find the *c.g.* of the frustum.

Answers

36. In the triangle $ADEA$, Fig. 34, draw a line DH from the point D to the center H of the side AE. Also draw AF from A to the center of

DE. Assume the triangle made up of a series of strips parallel to the base *DE* and of the form *BC*. The *c.g.* of this small strip is at its center *S*, which by construction lies on the line *AF*. Hence the *c.g.* of the triangle must lie somewhere on the line *AF*. Again, divide the triangle into a series of small strips parallel to the line *AE* and of the form *MN*. The *c.g.* of these small strips will fall along the line *DH*. Therefore, the *c.g.* of the triangle will be at the point of intersection *G* of the two lines *DH* and *AF*.

The *c.g.* might also be found by assuming equal weights *W* suspended from the vertices *A*, *D* and *E* of the triangle and then taking the moments of these weights about the base of the triangle. The moment arm of the weight *W* at the point *A* will be the altitude of the triangle. Let *Y* equal the distance from the base *DE* to the *c.g.* located at point *G* and take moments about the base *DE*, thus

$$W \times \text{altitude} = 3 \times W \times Y$$

or

$$Y = \tfrac{1}{3} \text{ the altitude}$$

Hence *the center of gravity of any triangle is at a distance of one-third the altitude from any and all bases.*

37. Not always; as an example see Fig. 35, which shows the strap of a connecting-rod. The *c.g.* of the strap lies near the point *G*.

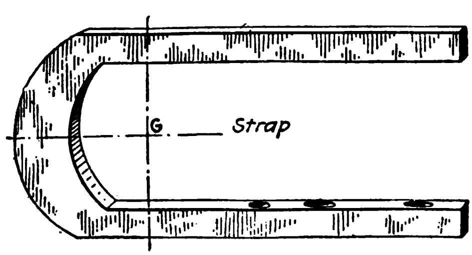

FIG. 35.

38. Before the plate was punched the *c.g.* was at the point *O* (see Fig. 36) or at the center of the diagonal *AB*. The punched plate is symmetrical with respect to the line *AB*. Assume the *c.g.* at the point *G* distance *X* in. from the point *O*. It can readily be shown that the distance from the point *O* to the center of the hole *C* is 2.12 in. The area of the punched plate is 225 − 12.57 = 212.43 sq. in. The moment about the point *O* of the *part cut away must equal the moment of the part remaining*. Thus

$$X \times 212.43 = 2.12 \times 12.57$$

or

$$X = \tfrac{1}{8} \text{ in. (approx.)}$$

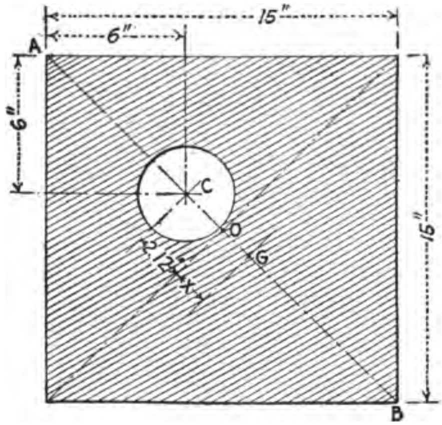

FIG. 36.

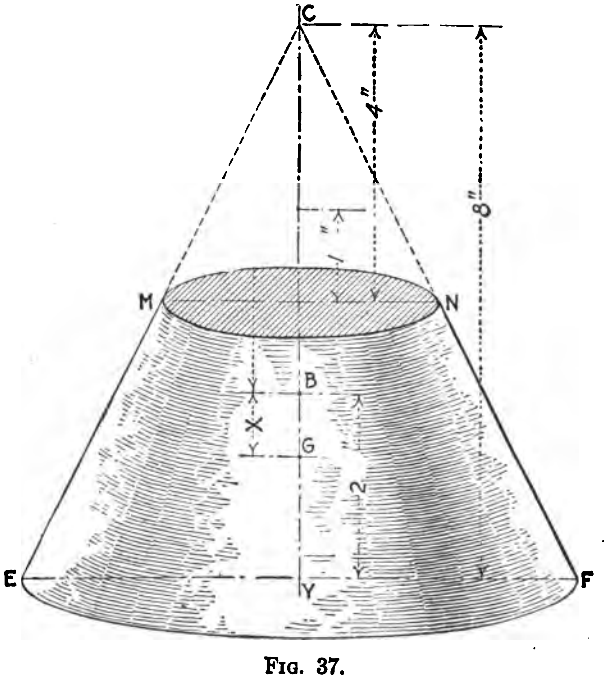

FIG. 37.

39. The weight of a 6-in. cast-iron ball is about 30 lb. and the weight of a 2-in. rod 18 in. long is 16 lb. The *c.g.* of the rod is 9 in. from the end, and the *c.g.* of the ball is 21 in. from the end, of the rod. Let X equal the distance from the end of the rod to the desired *c.g.* Take moments about the end of the rod, thus

$$9 \times 16 + 21 \times 30 = 46 \times X$$

or

$$46 \times X = 774$$

and

$$X = 16.8+ \text{ in.}$$

40. The *c.g.* will lie in the vertical axis CY of the frustum, Fig. 37. The *c.g.* of the cone from which the frustum was formed was one-fourth the altitude (2 in.) from the base EF or at the point B. The *c.g.* of the part of the cone which was cut away was 1 in. from the base MN. Let the *c.g.* of the frustum $MNFEM$ be located at the point G, X in. from the point B. To find the value of X take moments about the point B.

Thus,

$$16.76 \times 3 = 117.29 \times X$$

or

$$X = 0.43 \text{ in.}$$

Therefore, the *c.g.* of the frustum $MNFEM$ is on the line CY 1.57 in. from the base EF. (NOTE.—Volume of $MCNM$ = 16.76 cu. in.; volume of $MNFEM$ − 117.29 cu. in.)

CENTER OF GRAVITY—*Continued*

In problems where the body is symmetrical with neither the vertical nor the horizontal axes the following general method must be applied to find the *c.g.* Assume a given body to be divided into a large number of very small parts, and the weights of each part to be represented by the points w_1, w_2, w_3, etc., as shown in Fig. 38. Locate each of these weights from the axes OX and OY, thus w_1 is r_1 units from the OY axis, and y_1 units from the OX axis. The resultant of all these small weights will be the total weight W of the given body. Thus $W = w_1 + w_2 + w_3 + \ldots$, etc. This weight or resultant will act at the *c.g.* of the body. Let this point be X units from the OY axis and Y units from the OX axis. Then

the distance Y may be found by taking moments about the axis OX. Thus

$$w_1y_1 + w_2y_2 + w_3y_3 + \quad . \quad . \quad . \quad . = (w_1 + w_2 + w_3 + \quad . \quad . \quad .)$$
$$X\ Y = WY$$

or
$$Y = \frac{w_1y_1 + w_2y_2 + w_3y_3 + w_4y_4 + \ldots.}{w_1 + w_2 + w_3 + w_4 + \ldots.} \tag{11}$$

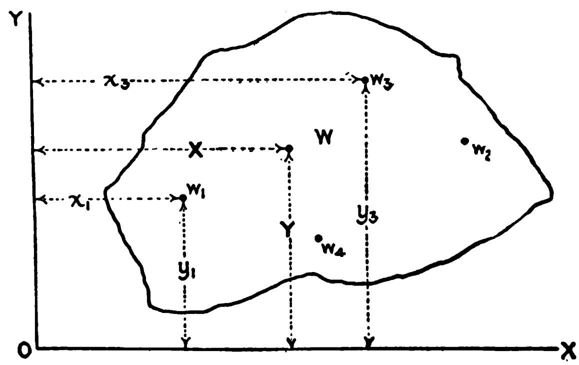

FIG. 38.

In like manner the value of X may be found by taking moments about the OY axis. Thus,

$$X = \frac{w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + \ldots.}{w_1 + w_2 + w_3 + w_4 + \ldots.} \tag{12}$$

If all the weights cannot be located in the plane of the OX and the OY axes, a third reference axis may be used and then,

$$Z = \frac{w_1z_1 + w_2z_2 + w_3z_3 + w_4z_4 + \ldots.}{w_1 + w_2 + w_3 + w_4 + \ldots.} \tag{13}$$

STABLE AND UNSTABLE EQUILIBRIUM

If a slight horizontal force P be applied to the cone shown in Fig. 39 so as to raise the point C, and then if the force P be removed it is evident that the cone will return to its original

position. When the force P is removed the weight W forms with the reaction at the point B, a couple which brings the cone back to its former position. If, however, the force P had been applied until the line GW fell outside of the point B, then the cone would have overturned, for in this case the weight W would form with the reaction at the point B a couple which would rotate the cone in a counter-clockwise direction which, of course, would overturn the cone.

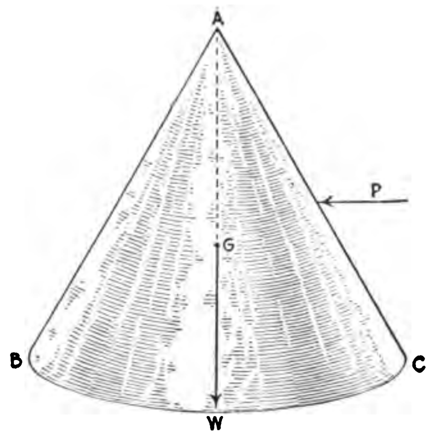

FIG. 39.

Therefore, a body is said to be in stable equilibrium when, if it be slightly displaced from its initial position, the forces acting on the body tend to bring it back to its original position; or the body is in unstable equilibrium if, after being slightly displaced, the forces acting tend to still further move it from its original position.

If, after the body is slightly displaced, the forces are in equilibrium, the body is said to be in neutral equilibrium. Thus in Fig. 39 when the force P is applied, the *c.g.* of the cone rises until the point G is in the same vertical line as the point B, then further application of the force P causes the *c.g.* to be lowered. Assume the cone to be resting on its slant height

AB, then any displacement of the body will neither lower nor raise its *c.g.* Hence, the following rule may be applied to test the kind of equilibrium: *If a slight displacement of a body raises its c.g. the body is in stable equilibrium, if the displacement lowers the c.g. the body is in unstable equilibrium, and finally if a slight displacement neither raises nor lowers the c.g. then the body is in neutral equilibrium.*

Still another test is this, if a vertical line drawn through the *c.g.* of the body falls within the plane of support the body is in stable equilibrium and if it falls outside the body is in unstable equilibrium.

Example.—A ball resting on the ground is in neutral equilibrium for its *c.g.* remains at a fixed distance from the ground. An engine bed resting on its foundation is in stable equilibrium, for if its *c.g.* be raised the bed will return to its position when the force is removed.

Study Questions

41. The one leg of a standard angle is 6 in. and the other leg is 4 in. If the thickness of the legs is ¾ in., locate the *c.g.* by using equations (11) and (12).

42. In what kind of equilibrium is a horizontal beam which is being raised by a rope attached at its center?

43. Give two illustrations of neutral equilibrium.

44. For the most stable equilibrium, should the *c.g.* of a coal truck be high or low?

45. The drum head of a B. & W. boiler rests on its curved surface. In what kind of equilibrium is it?

Answers

41. Divide the angle (see Fig. 40) into the two rectangles *OABCO* or *I*, and *DCFED* or *II*. The *c.g.* of rectangle *I* will be at the point *M*, which is the point of intersection of the two diagonals *AC* and *OB*. Likewise, the *c.g.* of the rectangle *II* is at the point of intersection *N* of the diagonals *CE* and *DF*. Draw the reference axes *OX* and *OY*, and assume the *c.g.* of the angle to be located at the point *G*, *X* in.

from the OY axis and Y in. from the OX axis. The area of the figure I is

$$\tfrac{3}{4} \times 4 = 3 \text{ sq. in.}$$

and the area of figure II is

$$\tfrac{3}{4} \times 5\tfrac{1}{4} = 3.94 \text{ sq. in.}$$

Then the total area of the angle is

$$3 + 3.94 = 6.94 \text{ sq. in.}$$

The point M is 2 in. from the OX axis and $\tfrac{3}{8}$ in. from the OY axis; also, the point N is $\tfrac{3}{8}$ in. from the OX axis and $3\tfrac{3}{8}$ in. from the OY axis. These areas may be substituted in equations (11) and (12) in place of w_1, w_2 and W since the weight of the angle is proportional to its area.

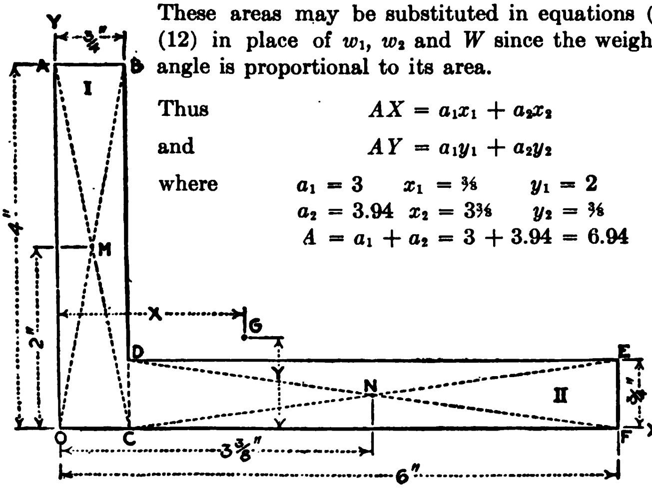

FIG. 40.

Thus $\quad AX = a_1x_1 + a_2x_2$

and $\quad AY = a_1y_1 + a_2y_2$

where $\quad a_1 = 3 \quad x_1 = \tfrac{3}{8} \quad y_1 = 2$

$a_2 = 3.94 \quad x_2 = 3\tfrac{3}{8} \quad y_2 = \tfrac{3}{8}$

$A = a_1 + a_2 = 3 + 3.94 = 6.94$

substituting these values in the above equations

$$X = \frac{a_1x_1 + a_2x_2}{A} = \frac{(3 \times \tfrac{3}{8}) + (3.94 \times 3\tfrac{3}{8})}{6.94} = 2.08 \text{ in.}$$

and

$$Y = \frac{a_1y_1 + a_2y_2}{A} = \frac{3 \times 2 + 3.94 \times \tfrac{3}{8}}{6.94} = 1.08 \text{ in.}$$

42. The beam is in unstable equilibrium.

43. A cylindrical tank lying on its side; (b) the spherical weight used on fly-ball governors, when the weight is resting on a horizontal surface.

44. The c.g. should be as low as possible so as to reduce the possibility of overturning the truck when rounding curves.

45. In neutral equilibrium.

CHAPTER VII

SUMMARY OF CONDITIONS OF EQUILIBRIUM

When any number of forces which are either concurrent or nonconcurrent and lie in the same plane, tend to produce a state of equilibrium it has been shown that certain general conditions must be satisfied by the forces. The first of these conditions is that there can be no resultant force. The second of the conditions is that there may be no resultant horizontal or vertical component, and for this to be true the algebraic sum of all the horizontal and the vertical components must equal zero. If the forces acting on the body pass through a common point the above conditions are sufficient to insure a state of equilibrium; but if the forces are nonconcurrent then the third condition, that the algebraic sum of the moments of all the forces about a given point must equal zero, is also necessary to completely determine the state of equilibrium. *A system of forces which is in equilibrium may produce either a state of rest, or a state of uniform motion of the body on which the system acts.* Hence the forces acting on a body which is in a state of uniform motion may be in equilibrium just as well as forces which act on a body at rest.

The following general statements will assist the student in summarizing the work of the preceding lessons and at the same time be an aid in the solution of problems dealing with Statics.

For equilibrium to exist when,

CASE I. Two forces act on a body.

The forces must be equal in magnitude, opposite in direction and have a common point of application.

CASE II. Three forces act on a body.

(a) *The forces must lie in the same plane.*

(b) *The lines of action of the forces pass through a common point.*

(c) *The three forces may be represented in magnitude and direction by the sides of a triangle taken in order.*

(d) *If the three forces are parallel the resultant force equals zero, and the algebraic sum of the moments of the forces about any given point equal zero.*

CASE III. Any number of forces act on a body.

(a) *If forces are concurrent and lie in the same plane they can be represented in direction and magnitude by the sides of a polygon taken in order.*

(b) *Any one force is equal and opposite to the resultant of all the other forces.*

(c) *The algebraic sum of the horizontal and the vertical components must equal zero.*

(d) *The algebraic sum of the moments of all forces about any given point must be zero.*

(e) *If the forces are parallel the rule* (*d*) *under Case III may be applied.*

It will be noted that in Case I the forces are necessarily concurrent. In Case II the forces may be either concurrent or parallel, and in Case III the forces may be concurrent, parallel or nonconcurrent, and *in all the cases the forces lie in the same plane.*

Problems coming under Case I are solved by making the two forces numerically equal to one another.

Problems coming under Case II are solved, either by applying the law of sines to the triangle of forces, or by the use of equations (8), (9) and (10).

Problems coming under Case II are solved by the aid of equations (8), (9) and (10).

The solution of any problem in Statics may be either analytical or graphical; that is, it may be solved by the use of equations as outlined above, or it may be worked out graphically on the drawing board. The solution of problems by the aid of scale diagrams gives rise to a special form of mechanics called "Graphical Statics," which is particularly applicable to the determining of the loads carried by the various members of roof and bridge trusses, cranes, hoists, etc. While in this course no attempt will be made to discuss "Graphical Statics" it might be well to state that this subject is simply an application of the polygon of forces.

The following study questions are intended to cover the three cases just discussed, to review the work of the previous chapters; further to test the student's ability to analyze his problem, to determine the best method of solution, and last, but not least, to test the student's ability to actually work problems of a practical nature.

Study Questions

46. A boiler stay-bolt makes an angle of 25 deg. with the shell. If the steam pressure is 120 lb. per sq. in. and the stay supports an area 6×9 in., find the pull acting in the stay-bolt.

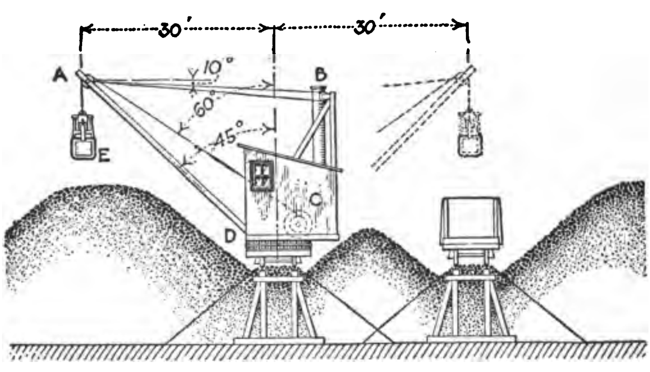

FIG. 41.

47. In Fig. 16 (see Chapter III) let the weight $W = 1000$ lb. and act midway between the points A and B. If the angle $a = 30$ deg., find the tension in the rod AC.

48. In problem 47, determine the direction and magnitude of the pressure at the point B.

49. In the locomotive crane shown in Fig. 41 the total load acting at the point A, due to the weight of the bucket, coal and boom is 4000 lb. The pull in the rope AC is 3500 lb. Find the force tending to crush the boom AD.

50. In problem 49, find the pull in the tie rope AB.

Answers

46. The pressure on the part of the boiler head supported by the stay is

$$6 \times 9 \times 120 = 6480 \text{ lb.}$$

and this force acts at an angle of 90 deg. to the head. The sides of the triangle $ABCA$ (see Fig. 42) represent the three forces acting at the point D where the stay is fastened to the head of the boiler. If the stay was

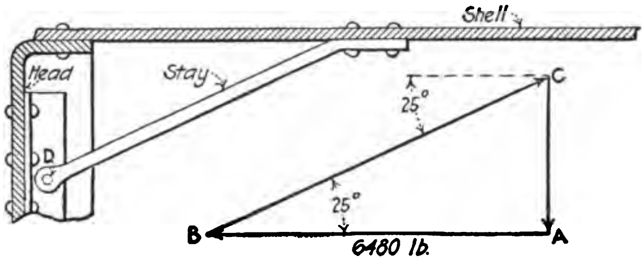

FIG. 42.

horizontal the pull would be 6480 lb., but since the stay makes an angle with the shell the pull becomes correspondingly greater. Let the side AB equal 6480 lb., or the force tending to blow out the 6 × 9-in. piece of the boiler head. The side BC will give the load on the stay and the side AC will represent the force tending to shear the rivets.

$$BC = \frac{BA}{\cos 25 \text{ deg.}} = \frac{6480}{0.906} = 7152$$

Therefore, the pull on the stay-bolt is 7152 lb.

47. Let the weight W be placed at the point E, midway between the points A and B (see Fig. 43). A vertical line drawn through the point E will bisect the line AC at the point D. The vertical force acting at the point A and at the point B will equal ½ W, since W is located at

the center of the line AB. Consider the point A as a free body, as shown in Fig. 43. The side MN of the triangle $MNOM$ is drawn horizontal and represents the thrust in the strut AB. The side ON is drawn vertical and equals ½ W while the side OM is drawn parallel to AC and represents

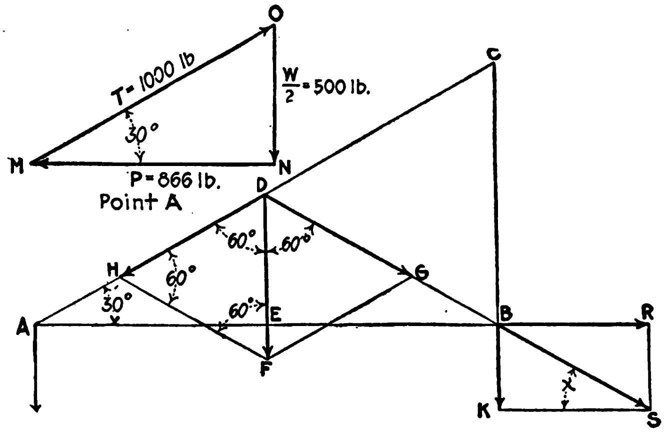

FIG. 43.

the pull in the tie-rod AC. The angle OMN = 30 deg. Therefore, the side OM or

$$T = \frac{\frac{1}{2}W}{\sin 30 \text{ deg.}} = \frac{500}{0.5} = 1000 \text{ lb.}$$

Likewise the side MN or

$$P = 1000 \times \cos 30 \text{ deg.} = 1000 \times 0.866 = 866 \text{ lb.}$$

Hence the tension in the tie-rod is 1000 lb., which is just one-half of what it was when the weight W was placed at the end of the bar AB.

48. There are three forces acting at the point B, namely, the vertical force ½ W, the horizontal pressure in the strut or 866 lb., and the reaction of the pin B. The resultant of the horizontal pressure in the strut and the vertical load at the point B is the pressure acting on the pin B. Construct the parallelogram $BRSKB$ (see Fig. 43) with the side BR = 866 lb., and the side BK = 500 lb. The diagonal BS gives the direction and magnitude of the pressure acting on the pin B. Thus

$$\begin{aligned} BS^2 &= RS^2 + BR^2 \\ &= 500^2 + 866^2 \text{ (since } RS = BK = \tfrac{1}{2}W) \\ &= 1{,}000{,}000 \end{aligned}$$

or $$BS = 1000 \text{ lb.}$$

Also the $\sin x = \frac{500}{1000} = 0.5$, or angle x = 30 deg.

Therefore, if the diagonal BS is extended backward it will pass through the point D since the angle $DBE = 30$ deg. = the angle x. It is to be noted that the point D is the point of intersection of the line of action of the weight W and the line of action of the pull in the tie-rod AC. Hence the direction of the pressure on the pin B might have been determined directly by drawing a line through the points D and B, as explained in Chapter III, under the solution of problem 20.

Still another method of finding the magnitude of this pressure is as follows: Construct the parallelogram $DHFGD$, letting the diagonal DF = the weight W (to scale). Then the side DH represents the tension in the tie-rod AC, and the side DC represents the pressure on the pin B. It is evident that the triangle $DFHD$ has all its sides equal since all its angles are equal, or $DH = DG = DF = 1000$. Therefore both the tension in the tie-rod and the pressure on the pin B are equal to 1000 lb. The force producing equilibrium at the point B is the reaction of the pin B, which is equal and opposite to the pressure BS, shown in Fig. 43.

49. There are four forces acting at the point A (see Fig. 41), namely, the weight of 4000 lb. acting downward, the pull of 3500 lb. in the hoisting cable AC, the unknown pull in the tie rope AB, and the unknown thrust in the boom AD. In Fig. 44, let OG equal the weight of 4000 lb. and let OH represent the pull of 3500 lb. in the hoisting cable; let OR represent the direction of the resistance offered by the boom, and OS the direction of the resistance of the rope. Under the action of these four forces the point A remains at rest and the forces are in equilibrium. Hence the direction and magnitude of the forces may be represented by the sides of a polygon taken in order. Now construct a force polygon by drawing the line OG vertical and making it equal 4000 lb. to an assumed scale; from the point G draw the line GK parallel to the force OH (which is the tension in the hoisting cable) and equal to 3500 lb. to the same scale as OG; from the point K draw the line KM parallel to the force OS (which is the unknown pull in the tie rope). Now extend the line RO until it intersects the line KM at the point M. The line MO is then the closing line of the polygon $OGKMO$ and gives the direction and magnitude of the resistance exerted by the boom to counteract the load acting upon it. By measurement the line KM is found to equal 3350 lb. and the line MO equals 8950 lb.

This problem may also be solved analytically by resolving all the forces into their horizontal and vertical components, as shown in Fig. 44. Thus OC is the horizontal component, and OD the vertical component of the force OS. Next apply the laws of equilibrium as stated in Chapter VII, under Case III. The ΣH and ΣV must equal zero, thus,

$$\begin{aligned}\Sigma H &= OS \cos 10 \text{ deg.} + OH \cos 30 \text{ deg.} - OR \cos 45 \text{ deg.} = 0\\ &= 0.985\, OS + 0.866 \times 3500 - 0.707\, OR = 0\end{aligned}$$

or $$0.707\ OR = 0.985\ OS + 3031$$

and $$\Sigma V = 0$$

$$\Sigma V = OR \sin 45 \text{ deg.} - OS \sin 10 \text{ deg.} - OH \sin. 30 \text{ deg.} - W = 0$$

or $$\Sigma V = 0.707\ OR - 0.174\ OS - 0.5 \times 3500 - 4000 = 0$$

or $$0.707\ OR = 0.174\ OS + 5750$$

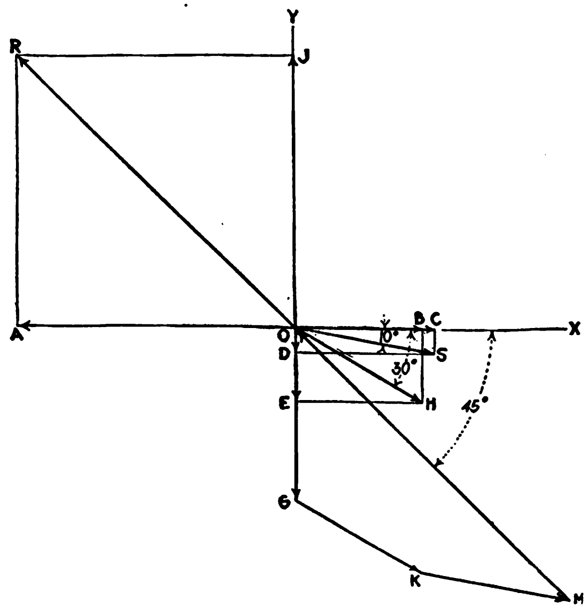

FIG. 44.

Hence $$0.985\ OS + 3031 = 0.174\ OS + 5750$$

or $$0.811\ OS = 2720$$

and $$OS = 3353 \text{ lb.}$$

Substituting this value of OS in the equation above and solving for OR there results,

$$0.707\ OR = 0.174 \times 3353 + 5750 = 6334$$

or $$OR = 8957 \text{ lb.}$$

Therefore the pressure on the boom is 8957 lb.

50. The value of the pull in the rope was found to be 3353 lb.

By this time, if the student has carefully followed the text and the solution of the various problems given he should be able to apply the principles studied to the solution of many problems that constantly face him in power-plant work. Lack of space forbids the solution of all the classes of problems that might arise.

The next few chapters will take up some of the applications of Statics to the simple machines, such as levers, balances, pulleys, etc.

As a final help to the student the following four rules are given for the solution of any and all problems either in Elementary Mechanics or any other subject.

I. *Think.*

II. *Analyze.*

III. *Equate.*

IV. *Solve.*

CHAPTER VIII

MACHINES

A machine is a device by means of which some form of energy is converted into useful mechanical work, or it may be a device whereby a small force may be used to balance a greater force without any work being done by either force. Only machines of the latter type will be discussed at present.

At some point of the machine a force is applied which is utilized in balancing a greater force at some other point in the machine. The force applied is generally spoken of as the *power* or the *effort,* and the force which is balanced is called the *weight* or the *resistance.* The term power, strictly speaking, means the time rate of doing work and as work does not enter into the present discussion the term effort will be used in the consideration of these elementary forms of machines.

LEVERS

Perhaps the simplest form of a machine is the *lever,* which consists of a bar resting on a fixed point or axis. The fixed axis is called the *fulcrum,* and the distance from the fulcrum to the points of application of the effort and the weight are called the arms of the lever. Thus Fig. 45 represents a lever commonly known as the crowbar or pinch bar. In this case the line DE represents the lever; the point C is the fulcrum; the force P is the effort and W is the weight or the resistance to be balanced. To find the relation between the effort P and the weight W, take moments about the fulcrum C. Thus

$$P \times B = W \times A$$

or

$$P = \frac{W \times A}{B} \tag{14}$$

The reaction R on the fulcrum of the lever is equal to the sum of the effort and the weight, or expressed as an equation,

$$R = P + W \tag{15}$$

Example.—In Fig. 45, if the distance $A = 4$ in. and the distance $B = 48$ in., what weight W may be balanced by a

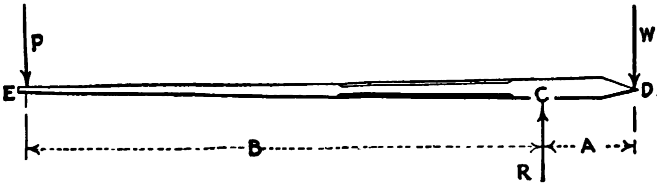

FIG. 45.

man exerting an effort of 75 lb. at the point E? This problem is solved by applying equation (14). Thus,

$$P = \frac{W \times A}{B} \text{ or } W = \frac{P \times B}{A} = \frac{75 \times 48}{4} = 900 \text{ lb.}$$

The reaction R on the fulcrum C is the sum of P and W or equals

$$900 + 75 = 975 \text{ lb.}$$

The ratio of the lengths of the arms of the lever is called the *mechanical advantage* of the lever, which, in the above case, is $^{48}/_{4} = 12$. If the length B were smaller than the length A there would be a *mechanical disadvantage.*

Levers are usually divided into three classes, depending upon the location of the effort and the weight relative to the fulcrum. In a lever of the *first class* the *effort P* and the *weight W* are on opposite sides of the fulcrum, as shown in Fig. 45.

In a lever of the *second class* the *effort P* and the *weight W* are on the same side of the fulcrum, the effort being the furthest from the fulcrum, as shown in Fig. 46, where P is the effort,

W is the weight and R is the reaction at the fulcrum, which, in this case, is the difference between the weight and the effort, or,

$$R = W - P \tag{16}$$

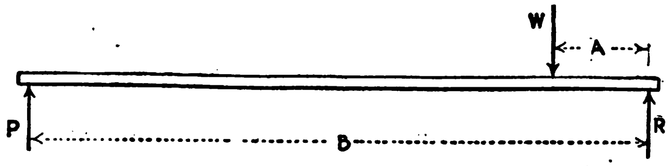

FIG. 46.

In a lever of the *third class the effort* and the *weight* are on the same side of the fulcrum, the weight being the furthest from the fulcrum, as shown in Fig. 47, where W is the weight, P is the effort, and the reaction R is the difference between the effort and the weight or,

$$R = P - W \tag{17}$$

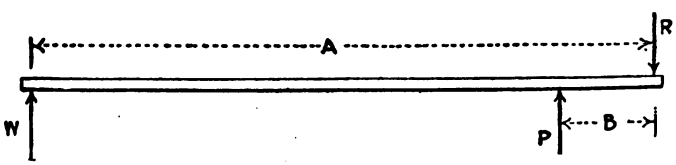

FIG. 47.

Illustrations of levers of the first class are the crowbar, a pair of shears, the oar of a boat, a claw hammer, etc. A good illustration of a lever of the second class is the wheelbarrow. Here the axle supplies the fulcrum; the effort is supplied by the man at the handles; the weight is the load plus the weight of the wheelbarrow assumed concentrated at the center of gravity of the two. A safety valve of the form shown in Fig. 48 is an example of a lever of the third class, for in this case the effort is the total pressure of the steam on the valve and the weight or resistance is the ball placed at the end of the lever. If the weight of a lever is

small compared to the forces acting on the lever, it may be neglected, but in cases such as a safety-valve lever its weight must be taken into account. This may best be illustrated by an example. Thus in Fig. 48 assume the diameter of the valve to be 3 in. and let $L = 30$ in., $C = 3$ in., the weight of the ball $W = 70$ lb., the weight of the lever, $w_1 = 12$ lb.

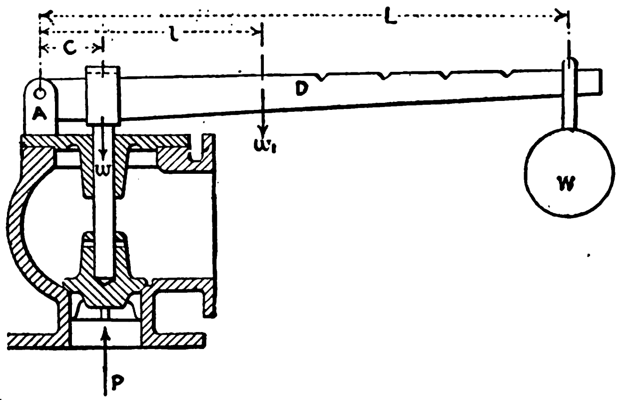

Fig. 48.

$l = 10$ in., and the weight of the spindle and valve, $w = 6$ lb. What steam pressure will be required to raise the valve from its seat?

Let p be the desired steam pressure in pounds per square inch. The area of the valve equals $0.7854 \times 3^2 = 7.07$ sq. in. and hence the total steam pressure P acting on the valve equals $7.07 \times p$. For equilibrium to exist the algebraic sum of the moments of all the forces about the point A must equal zero. Thus,

$$w \times C + w_1 \times l + W \times L = P \times C \tag{18}$$

or

$$6 \times 3 + 12 \times 10 + 70 \times 30 = 7.07 \times p \times 3$$

or

$$21.21\ p = 2238 \text{ and } p = 105.5 \text{ lb. per sq. in.}$$

Hence a pressure of not less than 105 lb. per sq. in will be required to raise the valve from its seat. In this problem the pressure or reaction at the pin A equals $P - w - w_1 - W = 746 - 6 - 18 - 70 = 652$ lb. As a general rule the distance l will be about one-third the distance L. However, if desired, the distance l may be determined by taking the lever apart and finding the point D at which the lever will balance.

Still another illustration of a lever of the third class would be the rocker-arm of the Corliss valve-gear of a horizontal type of engine. Here the fulcrum is located at the pin bearing of the lower end of the rocker-arm, the effort is the thrust of the eccentric rod, and the weight is the pull in the reach rod.

The various forms of bell cranks illustrate types of bent levers and the relation between the effort and the weight may be figured as above.

WHEEL AND AXLE

Another common form of machine is the *wheel and axle*, an example of which is the ordinary hoisting drum used

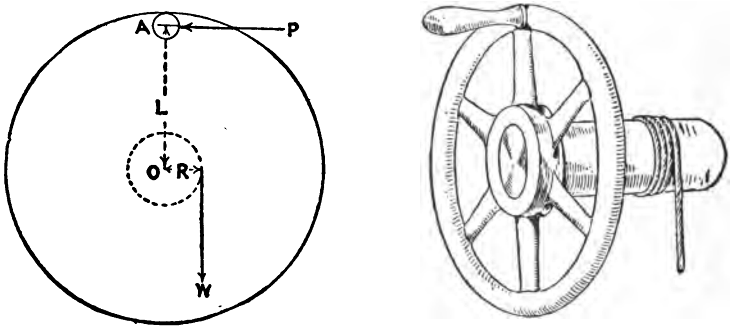

FIG. 49.

in various kinds of construction work. Thus in Fig. 49 let a force P be applied at the handle A of the drum or wheel to overcome the weight W acting at the circumference of the

axle whose radius equals R. To find the relation between P and W, take moments about the center O of the axle.

Thus

$$P \times L = W \times R \text{ or } P = \frac{W \times R}{L} \qquad (19)$$

Example.—What force P must be applied to overcome a weight W of 500 lb. acting at the circumference of a drum 8 in. in diameter, if the distance from the handle to the center of the axle is 20 in.? In this case $L = 20$, $R = 4$ and $W = 500$; therefore

$$P = \frac{W \times R}{L} = \frac{500 \times 4}{20} = 100$$

Study Questions

51. What weight may be balanced by the use of a crowbar if the fulcrum be located 3 in. from the *c.g.* of the weight, and a man exerts a force of 75 lb. at a distance of 48 in. from the fulcrum?

52. The distance from the axle of a wheelbarrow to the *c.g.* of the total load on the wheelbarrow is 12 in. The distance from the *c.g.* of the load to a point on the handle where a man applies a pull of 60 lb. is 40 in. What load can be balanced by the man?

53. In the above problem, find the pressure of the wheel on the ground when the man supports the handles.

54. In Fig. 48, $C = 4\frac{1}{2}$ in., $l = 15$ in., $w_1 = 25$ lb., $w = 10$ lb. and the diameter of the valve is 4 in. If the steam pressure in the boiler is 80 lb. per sq. in., at what point must a ball weighing 100 lb. be placed so as to prevent the valve from lifting?

55. What weight W may be balanced by the application of a force P of 80 lb. when $L = 18$ in. and $R = 3$ in. (see Fig. 49). Neglect all losses.

Answers

51. In this case the distance A (see Fig. 45) is 3 in.; the distance B is 48 in., and the effort P is 75 lb. Therefore,

$$W = \frac{P \times B}{A} = \frac{75 \times 48}{3} = 1200 \text{ lb.}$$

52. The wheelbarrow is a lever of the second class; hence in Fig. 46 the distance A is 12 in., the distance B is $(40 + 12) = 52$ in., and the effort $P = 60$ lb. Hence,

$$W = \frac{P \times B}{A} = \frac{60 \times 52}{12} = 260 \text{ lb.}$$

which would be the load that could be balanced by an effort of 60 lb.

53. The pressure of the wheel on the ground would be the reaction R of the fulcrum as shown in Fig. 46, or $R = (W - P) = (260 - 60) = 200$ lb.

54. The total pressure of the steam on the valve equals

$$0.7854 \times 4^2 \times 80 = 1005 \text{ lb.}$$

The distance L may be found from equation (18), thus,

$$w \times C + w_1 \times l + W \times L = P \times C$$

where $w = 10$ lb., $C = 4\frac{1}{2}$ in., $w_1 = 25$ lb., $l = 15$ in., $W = 100$ lb. and $P = 1005$ lb. Hence,

$$W \times L = P \times C - w \times C - w_1 \times l$$

or

$$L = \frac{P \times C - w \times C - w_1 \times l}{W}$$

$$= \frac{1005 \times 4.5 - 10 \times 4.5 - 25 \times 15}{100} = 41 \text{ in.}$$

therefore, to balance the steam pressure of 80 lb. per sq. in., the ball W weighing 100 lb. would be placed 41 in. from the fulcrum A.

55. Equation (19) may be used in the solution of this problem. Thus,

$$P = \frac{W \times R}{L}$$

or

$$W = \frac{P \times L}{R} = \frac{80 \times 18}{3} = 480 \text{ lb.}$$

It might be well to note at this point that the force of 80 lb. would not raise the weight of 480 lb. until the weight was set in motion by a force sufficient to overcome its inertia.

PULLEYS

The *pulley* is a machine operated by a flexible cord passing over a *grooved wheel,* which is mounted in a frame called the *block.* Pulleys are either *fixed* or *movable,* depending upon whether they are held in a fixed position or move with the given load. In this discussion, both the weight of the pulley and of the rope will be neglected as these are usually very small compared with the loads handled. Further, the friction at the bearings will be neglected, and it will be assumed that *the pull in a flexible cord or rope is uniform throughout its entire length.*

There are three systems of pulleys, but only the one in common use around the power plant will be discussed. A single fixed pulley has no mechanical advantage and can only change the direction of the pull on a given weight. Thus in Fig. 50, the effort P and the weight W are equal and this relation holds true irrespective of whether the pull P is vertical as shown by the line AC, or inclined as shown by the line AB.

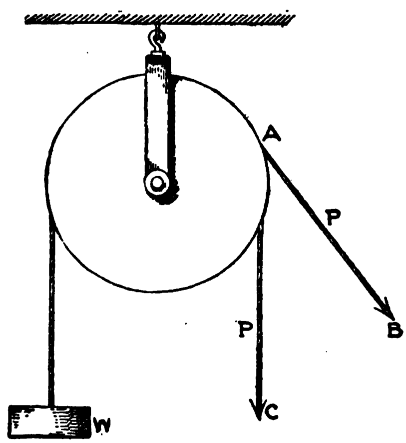

FIG. 50.

A single movable pulley A and a fixed pulley B may be used to considerable advantage. In Fig. 51, the rope is attached at the point C, passes over the movable pulley A and thence over the fixed pulley B to a point where the pull P is applied. The tension throughout the entire rope is P lb. Therefore, the total upward pull on the pulley A is $2P$ lb. and for equilibrium $2P$ must equal W or $P = \frac{W}{2}$, which means that a force of 50 lb. applied at the point D is capable of balancing a force of 100 lb. at the point A.

When a pulley is made up of a number of wheels mounted on a common axis, each wheel is called a "*sheave*." Usually, there will be one movable block and one fixed block, each

consisting of the same number of sheaves. A continuous rope passes over all the sheaves as shown in Fig. 52. The rope is secured to the bottom of the fixed pulley, from where it passes to the first sheave on the movable block, thence to the fixed pulley and so alternates until it finally leaves the last sheave on the fixed pulley, and is conducted to the point where the

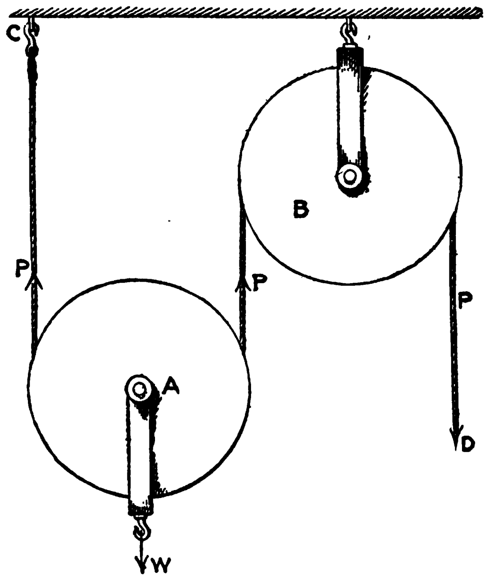

Fig. 51.

effort P is applied. Thus, if there are three sheaves in the movable block, the total upward pull on the block will equal $6 \times P$, since the pull in the rope is assumed uniform throughout its entire length, and there are six ropes supporting the movable block.

Hence the total upward pull on the movable block in a system of pulleys of this kind will equal the number of ropes attached to, or passing over the movable block, times the effort P on the rope. If one end of the rope is attached to the movable block, the latter will contain one less sheave than the fixed block and the

mechanical advantage will be less than that of the system shown in Fig. 52.

Another practical application of the principle of moments

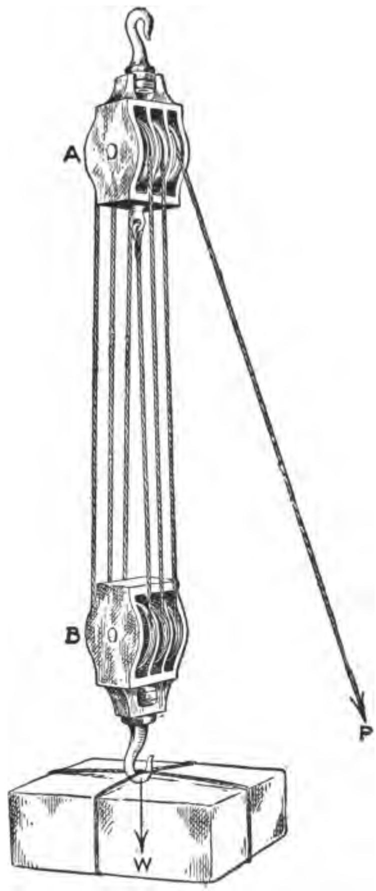

Fig. 52.

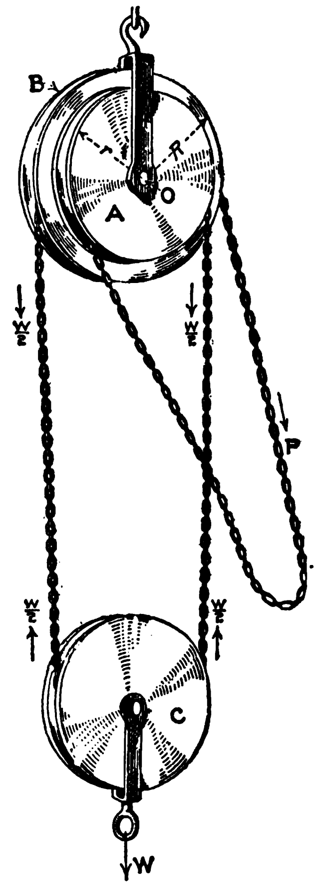

Fig. 53.

is in the *Weston Differential Pulley,* which consists of a fixed block containing two grooved sheaves of slightly different diameters, and a single movable pulley which is also grooved

so as to receive the links of a chain which passes around the sheaves as shown in Fig. 53. Any motion of the chain causes a positive motion of the sheaves as there can be no slipping of the chain on the sheaves. The relation between the effort P and the weight W may be determined as follows: In Fig. 53, let r be the radius of the smaller sheave on the fixed block and R_1 the radius of the larger sheave on the fixed block. Let P be the effort applied to the hoisting chain and W the weight to be balanced. It is evident that the pull in the chain passing over the movable sheave is $\frac{1}{2} W$ since there are practically two chains supporting the given weight. For equilibrium to exist, the algebraic sum of the moments of all the forces about the point O (which is the center of the fixed block) must equal zero. Thus,

$$P \times R + \tfrac{1}{2} W \times r - \tfrac{1}{2} W \times R = O$$

or

$$\tfrac{1}{2} W \times (R - r) = P \times R$$

hence,

$$W = 2P \times \frac{R}{(R - r)} \tag{20}$$

The *mechanical advantage* of the differential pulley is

$$\frac{2R}{(R - r)}$$

By making R and r as near equal as possible, the weight W may be much greater than the effort P. The efficiency of this hoist and the relative distances moved by the weight and the effort will be discussed fully under the lesson on work and power which will come later in the course.

Study Questions

56. In Fig. 51, what weight W could be balanced by a man who exerts a pull of 80 lb. at the point D?

57. In a system of pulleys arranged as in Fig. 52, there are four sheaves on the movable block. If one end of the rope be attached to the fixed block, what weight W could be balanced by an effort of 100 lb. being applied at the free end of the rope?

58. In problem 57, assume the rope attached to the movable block. How many sheaves would be on the fixed block if there are four on the movable block? What weight W might be balanced by an effort of 100 lb. being applied at the free end of the rope?

59. In problems 57 and 58 find the reactions on the hook supporting the fixed block.

60. In Fig. 53 if R = 8 in., r = 7 in., find the weight W that could be balanced by an effort of 100 lb., applied to the chain.

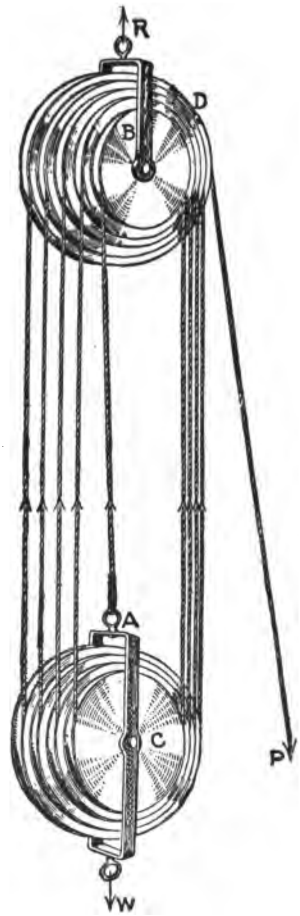

FIG. 54.

Answers

56. There are two ropes passing over the pulley A, Fig. 51, and the tension in each rope is 80 lb. Hence the weight that could be placed on the pulley A would equal

$$80 \times 2 = 160 \text{ lb.}$$

57. Since there are four sheaves on the movable block, Fig. 52, there would be $4 \times 2 = 8$ ropes passing over the movable pulley. The pull in each rope is 100 lb. Therefore, the weight W would equal

$$8 \times 100 = 800 \text{ lb.}$$

58. There would be five sheaves on the fixed block as will be evident from a careful inspection of Fig. 54 where for the sake of clearness the sheaves are made of varying diameter. The rope is attached to the movable block at the point A, passes over the first sheave B of the fixed block, thence to the sheave C of the movable block and so on until the rope leaves the last sheave D on the fixed block. There are nine ropes supporting the movable block and hence the weight W equals

$$9 \times 100 = 900 \text{ lb.}$$

59. In problem 57 the reaction R on the hook equals the sum of the weight W and the effort P or

$$R = 800 + 100 = 900 \text{ lb.}$$

In problem 58 the reaction R on the hook is

$$900 + 100 = 1000 \text{ lb.}$$

60. From equation (20)

$$W = \frac{2 \times P \times R}{(R - r)}$$

where $P = 100$ lb.;
$R = 8$ in.;
$r = 7$ in.

Hence

$$W = \frac{2 \times 100 \times 8}{(8 - 7)} = 1600 \text{ lb.}$$

CHAPTER IX

FRICTION

When an attempt is made to slide one body over another a certain resistance is encountered which must be overcome before the body will move. This force which tends to prevent the motion of one body on another is called the *force of friction* or simply the *friction* between the bodies. The amount of this friction will depend upon the nature of the surfaces in contact, and the materials of which the bodies are made. No matter how smooth or regular the surfaces in contact, this force of friction will always be present to counteract the forces tending

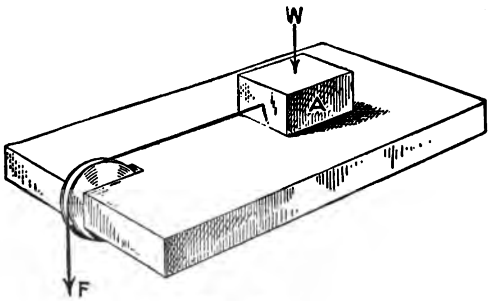

FIG. 55.

to move the bodies. Thus in Fig. 55 let the body A weighing W pounds rest upon a table. Attached to the weight A is a cord passing over the pulley shown. Let the force F be increased until the body A is just on the point of moving. It must be evident that the horizontal pull in the cord is equal to the force F, and hence the resistance between the body A and the table is equal to the force F and acts in a direction opposite to this force. The *force of friction will always act in a direction*

opposite to that in which the body tends to move. When the body A is just on the point of moving the force F is called the *limiting* or *static friction* between the body and the table. The force required to keep the body in a state of uniform motion is found by experiment to be less than the force which is required to put the body in motion. However, when the body is once in motion the force of friction between the body and the table is called the *sliding friction* and in all cases this will be less than the static or limiting friction.

In Fig. 55 it will be noted that the weight W is acting at right angles to the direction of motion of the body A. *The force or pressure which acts at right angles to the direction of motion of a given body is called the normal pressure between the given body and the surface over which it tends to move.* The greater this normal pressure the greater will be the friction between the given bodies, and the greater will be the force F required to move the body A in Fig. 55.

The ratio between the force of friction, or the force which is just great enough to put a given body in motion, and the normal pressure is called the *coefficient of friction of rest.* Thus, let

F = Force required to put the body in motion;
N = Normal pressure between the given bodies;
f = Coefficient of friction of rest.

Then from the above definition

$$f = \frac{F}{N} \tag{21}$$

or

$$F = f \times N \tag{22}$$

If a force of 50 lb. will move a weight of 200 lb. the coefficient of friction f will equal

$$\frac{F}{N} = \frac{50}{200} = 0.25$$

If, in equation (21), F represents the force that will produce uniform motion of the body A along the table, then the value

of f will be called the *coefficient of friction of motion,* and this is the value commonly given in tables.

A simple apparatus for determining this coefficient is shown in Fig. 56, which consists of a smooth plate CB of a given material, on which is placed a body A of some other material. The wedge D is forced under the plane CB until a slight tap on the weight A starts it down the plane with *uniform motion.* The angle BCX = angle a is then carefully measured. Let the line RS represent to scale the given weight W, and let this weight be resolved into two components RT and TS, the former

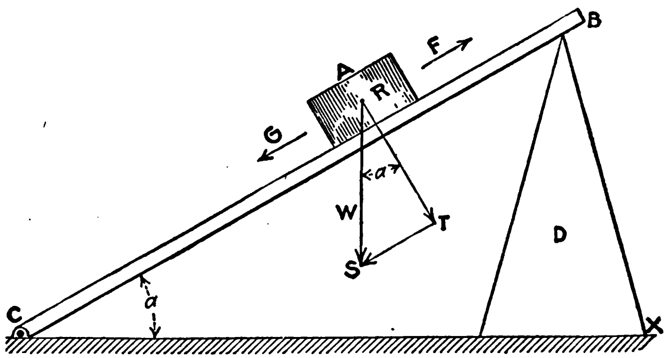

FIG. 56.

being at right angles to the plane CB and the latter being parallel to the plane. The effective force causing motion down the plane is therefore the component TS, which is the force of gravity acting parallel to the plane. The component RT causes no motion, but is the normal pressure between the weight W and the plane CB. This normal pressure N produces a force of friction F which acts parallel to the plane and tends to hold the body A at rest. Now from equation (22) the force of friction F is equal to the product of the coefficient of friction times the normal pressure N. For equilibrium to exist the force of gravity G acting down the plane must equal the force of friction F, which is acting up the plane or $F = G$. The angle SRT is equal to the angle BCX or the

angle a, since the line RT is perpendicular to the line CB, and the line RS is perpendicular to the line CX. Also the component $TS = W \times \sin a = G$ and the component $RT = W \times \cos a = N$. Therefore, since $F = G$,

$$W \times \sin a = W \times \cos a \times f$$

or

$$f = \frac{W \times \sin a}{W \times \cos a} = \tan a \qquad (23)$$

[NOTE.—The sine of an angle divided by the cosine of the angle equals the tangent of the angle.]

Hence the coefficient of friction f is the tangent of the angle a at which the plane CB must be inclined to produce uniform motion of the body A down the plane. The angle a is called the *angle of friction*. The force of friction F is practically independent of the area of the surfaces in contact.

Study Questions

61. A cast-iron body weighing 200 lb. rests upon a horizontal steel plate. If the coefficient of friction is 0.28, what force will be necessary to move the casting?

62. Is the force necessary to raise a body from the ground the same as the force required to move the body along the ground? Give a reason for your answer.

63. During a given experiment with the apparatus shown in Fig. 56 the angle a was found to be 15 deg. What was the coefficient of friction between the plane and the body A?

64. A railway car weighing 60,000 lb. requires a force of 600 lb. to keep it in a state of uniform motion along a horizontal track. What is the coefficient of friction between the wheels and the track, neglecting the friction at the journals?

65. At what angle would it be necessary to incline a given plane so that a given body whose coefficient of friction was 0.3 would move down the plane with a uniform speed?

Answers

61. The force of friction will equal

$$200 \times 0.28 = 56 \text{ lb.}$$

which will be the force that will keep the casting moving at a uniform

rate. The force required to move the casting will be greater depending upon the time it takes to get the casting in motion.

62. The force required to raise the body from the ground is greater than that necessary to move the body along the ground. The force of friction is the only force to be overcome in moving the body along the ground, while to lift the body the force of gravity (or the weight of the body) must be overcome. Generally the force of friction will be less than the weight of the body.

63. The coefficient of friction is equal to the tangent of the angle a, which in this problem is 15 deg.

Hence the value of f is

$$\tan 15 \text{ deg.} = 0.268$$

64. The coefficient of friction

$$f = \frac{\text{the force of friction}}{\text{the total normal pressure}}$$

In this problem the friction is assumed as 600 lb. Therefore,

$$f = \frac{600}{60{,}000} = 0.01$$

65. The tangent of the angle a made by the plane will equal the coefficient of friction. Thus

$$f = \tan a, \text{ hence } \tan a = 0.3$$

From a table of tangents the value of a is found to be 16 deg. 40 min.

FRICTION—*Continued*

Friction is encountered in every type of machine or apparatus used in the power plant. The aim of the engineer is to reduce it to a minimum by using bearing metals and lubricants having a low coefficient of friction, combined with other equally important conditions. The introduction of ball and roller bearings has reduced this coefficient of friction from 0.04 for the ordinary steel journal running in a brass bearing, to 0.003 for the better forms of roller bearings.

The following laws of friction were stated by Morin and Coulomb as a result of a series of experiments.

I. *The friction depends on the nature of the surfaces in contact, but is independent of the area in contact.*

II. *The friction varies directly as the normal pressure, assuming the materials of the surfaces in contact to remain the same.*

III. *The friction is independent of the velocity when there is sliding motion.*

Recent experiments have proved that these laws are not absolutely true, more especially in the case of law III, where it has been found that the *friction decreases when the velocity increases.* Also, as previously stated the force of friction when a body is just about to move, is greater than when the body is in motion. The subject of friction will be discussed more fully under the lessons relating to work and efficiency.

This closes the work on Statics which has had to do with forces that produced a state of equilibrium of a given body. The following chapters will deal largely with *forces that do not produce equilibrium,* but which tend to produce motion of various kinds; and also with forces that perform work in a more or less efficient manner.

CHAPTER X

PART II—KINETICS

MOTION

A body is said to be in *motion relative to another when its position changes with respect to that of the other body;* or as sometimes defined, *motion* is *simply displacement* or *change of position.* At the outset it is well to note that, in the strictest sense of the word, all motion is *relative,* and the term *absolute motion* is apt to be misleading. Take for illustration a train passing a certain station at a definite rate of motion, say 30 miles per hour. It is not fair to say that the *absolute motion* of the train is 30 miles per hour, for while that is the motion of the train relative to the station, it is also true that the station has a definite velocity relative to the center of the earth which in turn has its velocity relative to some point in the universe. As another example, consider the motion of the crosshead of a locomotive. Here the crosshead has a motion relative to the frame of the engine, it has still another motion relative to the crank-pin, a different motion relative to the track, and an entirely different motion relative to some fixed point in the universe. To avoid confusion it is best to compare the motions of various bodies to some point which for all practical purposes may be assumed as fixed. Thus to compare the motions of the crankpin and of the crosshead of the locomotive both might be referred to the frame of the engine. In the problems which are to follow the earth will be considered as the fixed point of reference.

As stated in Chapter V, motion may be divided into two general classes: (1) *Translation* and (2) *Rotation.* In the case

of translation all points of the given body will move in straight lines at the same rate of motion. In rotation, all points of the body will move in such a manner as to remain at a fixed distance from a given center of rotation.

Speed is the rate of motion of a body, or the rate at which the body describes its path. An automobile may be capable of developing a speed of 60 miles per hour and at the end of 2 hours be 120 miles from its starting point. The direction the machine went has nothing to do with its speed and hence the term speed involves only the two items of *magnitude* and *time,* without any reference being made to the *direction* of the motion. The speed of a body then is not sufficient to definitely fix the motion of the body.

Velocity is the rate of displacement of a body, and requires not only *magnitude* and *time,* but also *direction.* The velocity of a body completely specifies the motion of the body, as to direction, magnitude and time. To say that a body has a velocity of 50 ft. per sec. means that if the body were to move for one second at a uniform rate it would pass over 50 ft. of space. The velocity might change before the completion of the second and therefore the velocity of a body indicates the speed or rate of displacement *for a given instant only,* without any prediction as to what may be expected the next instant. The velocity of the body may be uniform or variable; hence motion may again be classified as *uniform* or *variable* depending upon whether or not the body covers the same amount of space for every second it is in motion.

The common unit for space is the *foot,* the unit for time is the *second* and the unit for velocity is the number of *feet per second.* Unless otherwise stated, these units will be used in the formulas that follow in the succeeding chapters.

The velocity of a rotating point may be expressed in *linear units,* that is, feet per second, or in *angular measure,* that is, in *revolutions per minute,* or *radians per minute.*

Example.—The distance from the center of the crankshaft to the center of the crankpin of a steam engine is 6 in.

If at a given instant the engine is running at 300 r.p.m., what is the velocity of the crankpin?

Solution.—This problem as stated is capable of two interpretations, depending upon what velocity is desired. It is evident that all points in the crank are making 300 r.p.m. Each time the engine makes one revolution the crank passes through 360 deg., or through 2π radians. (NOTE.—*The angle at the center of a circle which intercepts an arc on the circumference equal to the length of the radius of the circle is called a radian.*) The circumference of a circle is $2\pi R$, where $\pi = 3.1416+$; hence there are $\frac{2\pi R}{R} = 2\pi$ radians in the circumference. Therefore a

$$\text{radian} = \frac{360 \text{ deg.}}{2\pi} = 57.3 \text{ deg. (approx.)}$$

Since the engine is making 300 r.p.m., the crank will pass through

$$300 \times 2\pi = 600\pi \text{ radians}$$

This is known as the angular velocity of the crankpin. The second interpretation of the problem is to find the linear velocity of the crankpin, or the velocity with which the crankpin would move off in a straight line if, at any instant, the pin were suddenly released from the crank. The linear velocity will depend upon the distance of the pin from the center of the shaft. In one revolution the crankpin passes over a distance equal to the circumference of a circle whose radius is 6 in., and in 1 min. the crankpin moves over a distance equal to

$$\frac{2\pi \times 6 \times 300}{12} = 942 \text{ ft.}$$

Hence the linear velocity of the crankpin is 942 ft. per min., and the angular velocity is 600π radians per min.

The angular velocity of a body is equal to the linear velocity divided by the radius of rotation.

Let

R = The radius of rotation of the given body in feet;
N = Revolutions per minute (r.p.m.);
V = Linear velocity in ft. per min.;
a = Angular velocity in radians per min.

Then

$$V = 2\pi \times R \times N \tag{24}$$

and

$$a = 2\pi \times N \tag{25}$$

hence

$$\frac{a}{V} = \frac{2\pi \times N}{2\pi \times R \times N} \text{ or } a = \frac{V}{R} \tag{26}$$

The space S, traversed by a body moving with a uniform velocity of V ft. per sec. for a total time of T sec., is equal to $V \times T$ ft., or expressed as an equation

$$S = V \times T \tag{27}$$

Study Questions

66. At what crank angle is the friction between the crosshead and guide of an engine the greatest?

67. (a) What kind of motion has the crosshead of an engine, (b) the crankpin, (c) a point on the rim of the flywheel.

68. The stroke of an engine is 18 in. If the engine makes 250 r.p.m., what is the average speed of the piston in ft. per min.?

69. Is the velocity of the crosshead of an engine uniform or variable? Should the velocity of the crankpin be uniform or variable? State reason for your answer.

70. A fireman walks at the rate of 4 miles per hour. How long will it take him to pass through a boiler room which is 250 ft. long, assuming that he walks in a straight line?

Answers

66. The force of friction will be the greatest when the normal pressure between the crosshead and the guide is a maximum. This will occur when the crank makes an angle of 90 deg. with the horizontal, as shown in Fig. 10, where the normal pressure is given by the line ON, assuming cutoff not before half-stroke.

67. (a) The motion of the crosshead is variable, as it moves in one direction, then stops for an instant, and immediately moves in the opposite direction. Its motion might also be called *reciprocating,* since the direction of the motion changes with every stroke.

(b) The crankpin has uniform, rotary motion.

(c) A point on the rim of the flywheel has uniform, rotary motion. It might further be classified as *continuous* motion.

68. In one revolution of the engine the piston makes two strokes or moves through a distance of

$$2 \times 18 = 36 \text{ in., or } 3 \text{ ft.}$$

If the engine makes 250 r.p.m., the piston speed will equal

$$250 \times 3 = 750 \text{ ft. per min.}$$

69. The velocity of the crosshead is variable, since it changes from zero, when the engine is on dead center, to a maximum, when the crank and connecting-rod are nearly at right angles, and then back to zero. when the engine reaches the other dead center. The velocity of the crankpin should be uniform, especially in engines directly connected to generators, where uniform rotation is essential to constant voltage.

70. Four miles per hour is equivalent to

$$4 \times 5280 = 21{,}120 \text{ ft. per hr.}$$

which reduced to feet per minute is $\frac{21{,}120}{60} = 352.$

Therefore, since the fireman can walk at the rate of 352 ft. per min., it will take him $\frac{250}{352}$ = about $\frac{5}{7}$ of a min. to cross the boiler room.

COMPOSITION AND RESOLUTION OF VELOCITIES

A velocity, like a force, may be represented graphically in magnitude and direction by a straight line, drawn to a given scale, and having an arrow-head placed at the end of the line to indicate the direction of the velocity. Thus if the line OB (Fig. 57) is 2 in. long, and 1 in. represents 20 ft. per sec., the body A will have a velocity of 40 ft. per sec. in a direction of 30 deg. to the horizontal reference line OX.

Again, as for a force, a velocity may be resolved into any number of components. For convenience in dealing with problems, velocities will be resolved into their horizontal and

vertical components. Velocities acting to the right or upward will be considered as positive, and those acting to the left or downward as negative. *The component of a velocity is*

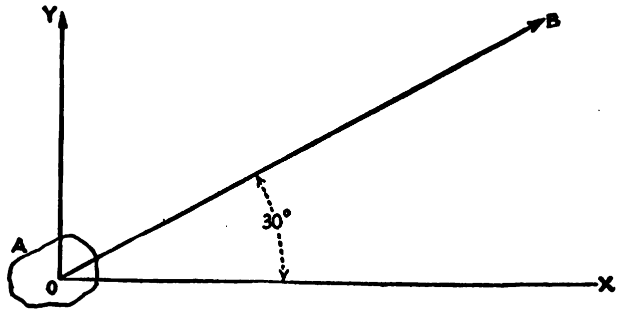

FIG. 57.

the rate at which the given body is traveling in the direction of the component. As an example, assume a train traveling due northeast at the rate of 40 miles per hour. Let this velocity be represented by the line *OB* (Fig. 58), making an angle of 45 deg. with the horizontal reference line *OX*. From the point *B*

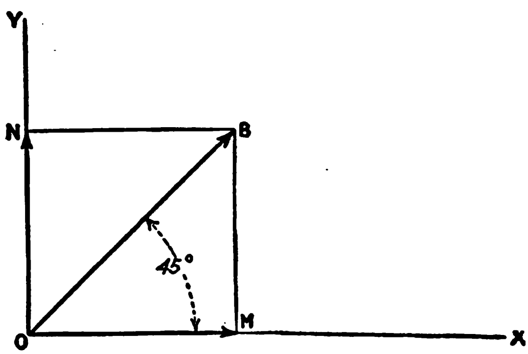

FIG. 58.

drop the perpendicular *BM*, forming the intercept *OM* on the line *OX*. In like manner find the intercept *ON*. Then the line *OM* will represent the rate at which the train is traveling

east, and the line ON will give the rate at which the train is traveling north. But

$$OM = OB \times \cos 45 \text{ deg.} = 40 \times 0.707 = 28 \text{ miles per hour}$$

Likewise

$$ON = OB \times \sin 45 \text{ deg.} = 40 \times 0.707 = 28 \text{ miles per hour}$$

which means that at the end of one hour the train will be 28 miles east of its original position and 28 miles north. During the hour it has gone 40 miles in the direction indicated by the line OB.

Formulas (1) and (2) given in Chapter I may be used for finding the horizontal and vertical components of a velocity.

RESULTANT VELOCITY

The resultant of two or more velocities is a single velocity that will produce the same motion of a body as the combined action of the other velocities. If the velocities are parallel and in the same direction the resultant velocity will equal the sum of the two velocities. Thus suppose the conductor of a train walks toward the forward end of the car at four miles per hour, and at the same time the train is moving at 36 miles per hour. When the conductor stands still the train carries him 36 miles per hour, but when, in addition, he walks four miles per hour, he has a velocity of $36 + 4 = 40$ miles per hour relative to the earth. If the conductor walks toward the rear of the train he will have a velocity of $36 - 4 = 32$ miles per hour relative to the ground.

To illustrate the method of finding the resultant of two velocities, take an overhead traveling crane in the turbine or engine room of any large power plant. Let the line OX (Fig. 59) represent the center line of the crane, and the line OY the direction the crane moves in passing down the room. Assume the crane as stationary, and suppose a workman crosses the girder of the crane at V_2 ft. per sec., or at the end of a certain number of seconds (say, T) he arrives at the point B. Again, suppose

the workman stands at the point O on the crane which is now moving down the room with a velocity of V_1 ft. per sec., and at the end of T sec. arrives at the point C. Now let the workman start at the point O and walk along the girder with a velocity of V_2 ft. per sec., and at the same time let the crane move down the room with a velocity of V_1 ft. per sec. At the end of T sec. the man arrives at B on the crane, but this point, due to the velocity of the crane has been moved to D. Therefore,

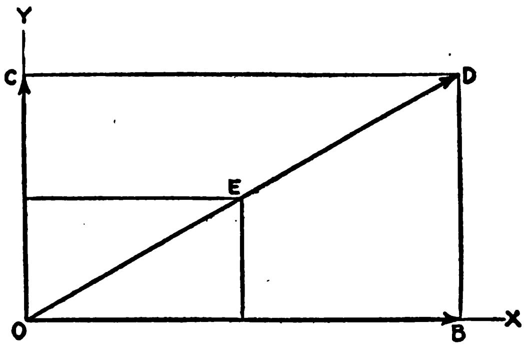

FIG. 59.

the man, due to the combined action of his own velocity and that of the crane, has arrived at D. If both man and crane had moved for $T/2$ sec., at the end of this time the man would be at E. It must be evident that the points D and E lie on the same straight line OD which is the diagonal of the parallelogram $OCDBO$. If another man started at the same time as the crane and walked in a direction OD with a velocity of OD ft. per sec., he would be directly under the man walking on the crane girder, overhead. In other words, the diagonal OD represents the resultant velocity of the combined velocities V_2 and V_1. This statement could be proved equally well if the angle COB was either acute or obtuse.

From the foregoing discussion the following law known as the parallelogram of velocities may be stated. *If two velocities*

acting on a body at the same time can be represented in magnitude and direction by the sides of a parallelogram, the resultant velocity will be represented in magnitude and direction by the diagonal of the parallelogram drawn through the point of intersection of the two sides representing the given velocities. Thus in Fig. 60, let the line OA represent a velocity of V_2 ft. per sec. and the

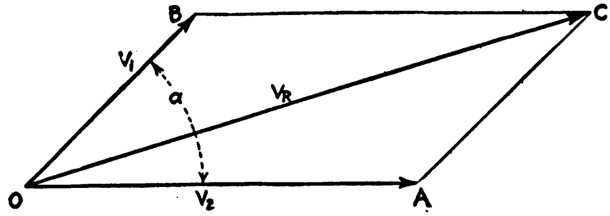

FIG. 60.

line OB making an angle of a deg. with the first velocity represent a second velocity of V_1 ft. per sec. The resultant velocity will be given by the equation,

$$V_R = \sqrt{V^2_1 + V^2_2 + 2 \times V_1 \times V_2 \times \cos a} \tag{28}$$

The proof for this equation is similar to the proof of equation (5) given in Chapter II.

A body may have several simultaneous velocities not necessarily in the same plane. For example, take a body being raised by a traveling crane. At the same time the body is moving upward with a given velocity, it may also have a velocity due to the motion of the crane across the shop, and another velocity due to the motion of the trolley across the girder of the crane. The velocity of the body relative to the floor of the shop is then the resultant of three velocities.

In Fig. 61 let the line OB equal the velocity V_1 with which the crane is traveling, the line OC equal the velocity V_2 with which the trolley is moving, and the line OD represent the velocity V_3 with which the body is being lifted. The diagonal OE is the resultant of the two velocities V_1 and V_2. This

resultant combined with the velocity V_3 gives the final resultant velocity OR, whose value is found as follows:

$$\overline{OE}^2 = \overline{OB}^2 + \overline{BE}^2 \text{ (since angle } COB \text{ is 90 deg.)}$$
$$= \overline{OB}^2 + \overline{OC}^2 \text{ (since } BE = OC\text{)}$$

also $$\overline{OR}^2 = \overline{OE}^2 + \overline{ER}^2 \text{ (since angle } OER \text{ is 90 deg.)}$$
$$= \overline{OE}^2 + \overline{OD}^2 \text{ (since } ER = OD\text{)}$$

hence $$\overline{OR}^2 = \overline{OB}^2 + \overline{OC}^2 + \overline{OD}^2 = V_1^2 + V_2^2 + V_3^2$$

and $$OR = \sqrt{V_1^2 + V_2^2 + V_3^2} \qquad (29)$$

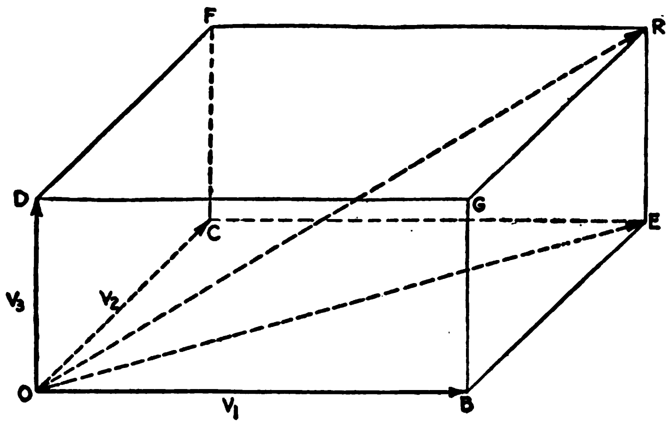

Fig. 61.

Study Questions

71. The stroke of an engine is 12 in. If the engine makes 300 r.p.m., find the velocity of the crosshead when the crank makes an angle of 45 deg. with the horizontal. The length of the connecting-rod is 30 in.

72. A man rows a boat with a velocity of six miles per hour. The stream flows at two miles per hour. At what angle must the man point his boat so as to land at a point directly opposite his starting place?

73. With what velocity will he cross the stream?

74. If the stream is 1000 ft. wide, how long will it take him to row across?

75. A load is being hoisted at 200 ft. per min., by a crane which is traveling at 50 ft. per min. At the same time the trolley is crossing the crane at 75 ft. per min. Find the velocity of the load relative to the ground.

Answers

71. The length of the crank OA (Fig. 62) is 6 in. In one revolution of the engine the crankpin A will move a distance equal to

$$2 \times 3.1416 \times 6 = 37.7 \text{ in.} = 3.14 \text{ ft.}$$

The velocity of the crankpin in feet per minute equals

$$300 \times 3.14 = 942 \text{ ft.}$$

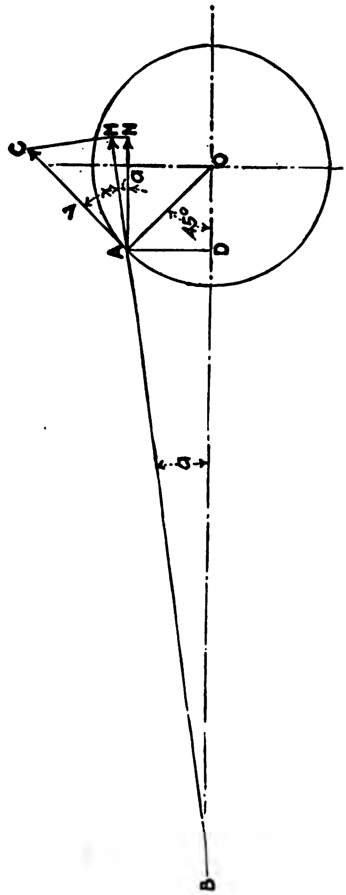

Fig. 62.

Let the line AC drawn at right angles to the line OA, represent this velocity. Extend the line of the connecting-rod BA and from C drop the perpendicular CM to AB. Then the line AM will give the velocity with which the connecting-rod is moving at the given instant and, since the rod is assumed rigid, the velocity of B (which represents the crosshead) must equal the velocity AM of the point A. The velocity of the crosshead is the horizontal component of the velocity of the connecting-rod. Therefore, draw the line AN horizontal, and from M drop the perpendicular MN to the line AN. The numerical value of the velocity AN may be found as follows:

$$\sin a = \frac{AD}{AB}$$

and

$$AD = OA \times \sin 45 \text{ deg.} = 6 \times 0.707 = 4.24 \text{ in.}$$

or

$$\sin a = \frac{4.24}{30} = 0.14$$

hence angle $a = 8$ deg. (approx.). By construction the angle CAN = (angle a + angle x) = 45 deg. Now, since the angle a equals 8 deg. it must be evident that the angle x equals $(45 - 8) = 37$ deg.

But

$$AM = AC \times \cos 37 \text{ deg.} = 942 \times 0.799 = 752$$

and

$$AN = AM \times \cos 8 \text{ deg.} = 752 \times 0.99 = 745$$

Therefore, the velocity of the crosshead at the given instant is 745 ft. per min.

This problem is inserted to illustrate the fact that it is possible for a single velocity to have a number of components acting in any number of directions. As a general rule the velocity of the crosshead will be determined graphically as shown in Fig. 62 instead of being figured numerically as has been done in this problem.

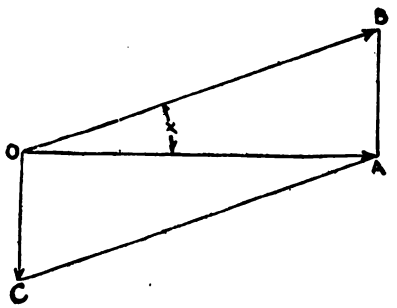

FIG. 63.

72. The resultant of the velocity with which the man is rowing the boat and the velocity of the stream must be a horizontal velocity if the boat is to land at a point directly opposite its starting point. In Fig. 63, let OA represent this resultant velocity, OB the direction the man has to head the boat and OC the direction of the current of the stream. From the conditions of the problem, OB equals six miles per hour and OC equals two miles per hour; hence

$$\sin x = \frac{AB}{OB} = \frac{2}{6} = 0.333, \text{ or } x = 19\tfrac{1}{2} \text{ deg.}$$

73. The velocity with which he is crossing the stream is given by the line OA (Fig. 63) which is equal to

$$OB \times \cos x = 6 \times 0.943 = 5.66 \text{ miles per hour} = 498 \text{ ft. per min.}$$

74. Time equals space divided by the average velocity. Or

$$\text{time} = \frac{1000}{496} = 2 \text{ min. (approx.)}$$

75. The desired velocity may be found by substituting the given velocities in equation (29). Thus $V_1 = 50$ ft. per min., $V_2 = 75$ ft. per min. and $V_3 = 200$ ft. per min. Hence, letting X equal the velocity of the load relative to the ground, there results,

$$X = \sqrt{50^2 + 75^2 + 200^2} = \sqrt{48{,}125} = 219 + \text{ft. per min.}$$

CHAPTER XI

UNIFORM AND ACCELERATED MOTION

Acceleration is the rate at which the velocity of a body changes and is expressed in the number of feet per second the velocity changes for every second the body is in motion. This acceleration may be *uniform* or *variable;* if uniform the velocity will increase at a uniform rate. *The formulas which are derived in this section apply only to problems involving uniformly accelerated motion.*

A body starting from rest and moving with an acceleration of a ft. per sec., at the end of T sec. will have acquired a velocity of aT ft. per sec. Let V = this velocity. Then

$$V = a \times T \tag{30}$$

The average velocity during the time the body is in motion will be the sum of the final and initial velocities divided by two or

$$\left(\frac{0 + V}{2}\right) = \left(\frac{0 + aT}{2}\right) = \frac{aT}{2}$$

The space S passed over by the body will equal the average velocity multiplied by the time, or

$$\frac{aT}{2} \times T = \tfrac{1}{2}\, aT^2$$

hence,

$$S = \tfrac{1}{2}\, aT^2 \tag{31}$$

Also, from equation (30),

$$T = \frac{V}{a}$$

Substituting this value for T in equation (31), thus

$$S = \tfrac{1}{2}\, a \times \left(\frac{V}{a}\right)^2 = \frac{V^2}{2a}$$

or

$$V^2 = 2\, aS \tag{32}$$

and

$$a = \frac{V^2}{2\,S} \tag{33}$$

Therefore, when a body starts from rest the acceleration may be found if the final velocity and the space passed over are known. Equation (33), it will be noted, is *independent of the time.*

Illustration.—A train starts from rest and after it has passed over a distance of 1000 ft. attains a speed of 22 ft. per sec. What is its acceleration?

From equation (33) $a = \frac{V^2}{2S}$, where V = the final velocity in feet per second, and S is the distance passed over during the attaining of the given velocity,

$$a = \frac{22 \times 22}{2 \times 1000} = 0.24 \text{ ft. per sec. per sec.}$$

In certain problems it may be desirable to express the acceleration in some other units as miles per hour per second, or miles per hour per hour.

If the body has an initial velocity of U feet per second before the increase in velocity takes place the formulas just stated will be somewhat modified. Thus, let U = the initial velocity of the body in feet per second. Then if the body moved with a uniform velocity only, for T sec. the space passed over would be UT ft. Now, if *in addition to the uniform velocity* the speed is being accelerated, then the space passed over during the time of the acceleration will be $\frac{1}{2} aT^2$, as shown in equation (31), to which must be added the space passed over due to the uniform velocity of U ft. per sec. Therefore, the total space passed over by a body moving with an initial velocity of U ft. per sec. which is accelerated at the rate of a ft. per sec. for a period of T sec., will be given by the equation,

$$S = UT + \frac{1}{2}aT^2 \tag{34}$$

The final velocity of the body at the end of the period of acceleration will equal the sum of the initial velocity U, plus the gain in velocity, which, from equation (30), is aT. Let V_1 = the final velocity, then,

$$V_1 = U + aT \tag{35}$$

Example.—A train is running at 15 miles per hour (22 ft. per sec.). The engineer turns on more steam so as to give an acceleration of 0.75 ft. per sec. (*a*) What is the velocity of the train at the end of 8 sec.?

From equation (35), $V_1 = U + aT$, where $U = 22$ ft. per sec., $a = 0.75$ ft. per sec. and $T = 8$ sec.; hence,

$$V_1 = 22 + 0.75 \times 8 = 28 \text{ ft. per sec.}$$

(*b*). How far did the train move while the gain in speed took place? From equation (34), $S = UT + \frac{1}{2}aT^2$ where $U = 22$ ft. per sec., $T = 8$ sec. and $a = 0.75$ ft. per sec.; hence,

$$S = 22 \times 8 + \tfrac{1}{2} \times 0.75 \times 8^2 = 200 \text{ ft.}$$

The average velocity during the time the body is in motion is $\frac{U + V_1}{2}$ and the space passed over in T sec. is S, whose value is

$$S = \left(\frac{U + V_1}{2}\right) \times T \qquad (36)$$

From equation (35), $T = \left(\frac{V_1 - U}{a}\right)$. Substitute this value of T in equation (36) and there results,

$$S = \frac{(U + V_1)}{2} \times \frac{(V_1 - U)}{a} = \frac{V_1^2 - U^2}{2a}$$

or

$$V_1^2 - U^2 = 2aS \qquad (37)$$

Equation (37) may be used to check the results of the above problem, thus the final velocity V_1 of the train was found to be 28 ft. per sec. and the initial velocity was 22 ft. per sec. and the acceleration 0.75 ft. per sec. Substitute these values in equation (37) and there results,

$$\overline{28}^2 - \overline{22}^2 = 2 \times 0.75 \times S$$

or

$$S = \frac{28^2 - 22^2}{2 \times 0.75} = \frac{784 - 484}{1.5} = \frac{300}{1.5} = 200 \text{ ft.}$$

If the velocity of a body is being decreased there will take place a deceleration or retardation in place of an acceleration and the value of a will become *negative* instead of *positive.* The formulas for retarded motion will be as follows:

$$S = UT - \tfrac{1}{2} aT^2 \tag{38}$$

$$V_1 = U - aT \tag{39}$$

$$V_1^2 = U^2 - 2aS \tag{40}$$

Study Questions

76. A subway train starting from rest attains a speed of 30 miles per hour (44 ft. per sec.) in a distance of 800 ft. What is the acceleration in feet per second per second?

77. The velocity of a moving body changes from 66 to 22 ft. per sec. while the body moves through a distance of 1000 ft., what is the retardation?

78. Express an acceleration of 2 ft. per sec. per sec. in miles per hour per second; in miles per hour per hour.

79. A train is traveling at 60 miles per hour (88 ft. per sec.). The engineer applies the brakes and at the end of 20 sec. the speed is reduced to 15 miles per hour (22 ft. per sec.). Find the deceleration.

80. How far did the train travel during the application of the brakes?

Answers

76. From equation (33)

$$a = \frac{V^2}{2S}$$

where a = The desired acceleration;
V = The velocity, 44 ft. per sec.; and
S = The space, 800 ft.

Hence,

$$a = \frac{44 \times 44}{2 \times 800} = 1.21 \text{ ft. per sec. per sec.}$$

77. From equation (40)

$$V_1^2 = U^2 - 2\ aS, \text{ or } 2\ aS = U^2 - V_1^2$$

where U = 66 ft. per second;
V = 22 ft. per second; and
S = 1000 ft.

Therefore,

$$a = \frac{U^2 - V^2}{2S} = \frac{66^2 - 22^2}{2 \times 1000} = 1.936 \text{ ft. per sec. per sec.}$$

78. An acceleration of 2 ft. per sec. per sec. means that for every second the body is in motion the increase or gain in velocity is 2 ft. per sec. Then the gain in velocity in feet per minute for every second the body is in motion would equal 60×2 or 120 ft. per min. per sec., which is to say that at the end of one minute the body would be moving with a velocity of 120 ft. per sec. Likewise the gain in velocity in feet per hour would equal 120×60 or 7200 ft. per hour per sec. Since there are 5280 ft. in a mile, 7200 ft. will equal $\frac{7200}{5280} = 1.36$ miles. Hence an acceleration of 2 ft. per sec. per sec. is equivalent to an acceleration of 1.36 miles per hour per sec. Now if the velocity is increasing at the rate of 1.36 miles per hour for every second the body is in motion, then for every hour the body is in motion the gain in velocity would be $3600 \times 1.36 = 4896$ miles per hour. Therefore, an acceleration of 2 ft. per sec. per sec. is equivalent to an acceleration of 4896 miles per hour per hour. This shows that the value of a given acceleration varies directly as the square of the factor changing the unit of time and inversely as the factor changing the unit of length.

79. From equation (39)

$$V_1 = U - aT \text{ or } a = \frac{U - V_1}{T}$$

In this problem

$$U = 88 \text{ ft. per sec.};$$
$$V_1 = 22 \text{ ft. per sec.};$$
$$T = 20 \text{ sec.}$$

Hence,

$$a = \frac{88 - 22}{20} = \frac{66}{20} = 3.3 \text{ ft. per sec. per sec.}$$

80. From equation (40)

$$V^2_1 = U^2 - 2aS \text{ or } S = \frac{U^2 - V^2_1}{2\,a} = \frac{88^2 - 22^2}{2 \times 3.3} = 1100 \text{ ft.}$$

UNIFORM AND ACCELERATED MOTION—*Continued*

Space may be represented graphically by an area, just as the work done in an engine cylinder is proportional to the area of the indicator diagram. It has previously been shown that the space traversed by a body is the product of the average velocity times the time; therefore, for an area to represent space, the base of the figure must represent time and the altitude of the figure represent the velocity of the body.

Thus, in Fig. 64, let the distances measured parallel to the axis OX be proportional to the time, and the distances measured parallel to the axis OY be proportional to the velocity of the body. Now suppose a body moves for an interval of time indicated by the line AB and during this time the velocity varies as shown by the irregular line $ACDEFB$, then the space passed over by the body will be given by the area $ACDEFBA$, for this area is equal to the product of the base AB (which corresponds to a certain interval of time) and the altitude

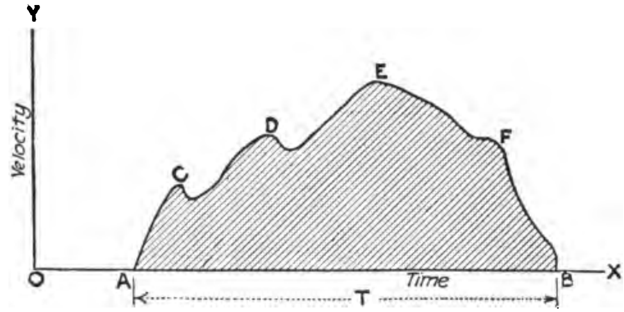

FIG. 64.

which is the average height of the figure and represents the average velocity. In a case like this the average velocity can be obtained by dividing the figure by vertical lines into 10 or 20 smaller areas, measuring the mean height of each and finding the average height of them all.

A graphical representation affords an easy method for becoming familiar with the formulas for uniform and accelerated motion. In Fig. 65 let the line AB represent a given interval of time of T sec.; let the line AC be the initial velocity U of the body in feet per second; and the line BE give the final velocity V_1 of the body. Draw the line CD parallel to AB. Evidently the line DE will be the gain in velocity during the interval of T seconds, since the line $DB = AC = U$. But this gain in velocity is equal to the acceleration times the time or aT. The figure, then, shows at a glance that the final

velocity V_1 is equal to the initial velocity U plus the gain in velocity which is aT, or

$$V_1 = U + aT$$

which is the same as equation (35).

If the body is moving with a uniform velocity only, the space traversed in T sec. will be equal to the area $ACDBA$ or UT. Likewise if the body starts with an initial velocity of U ft. per

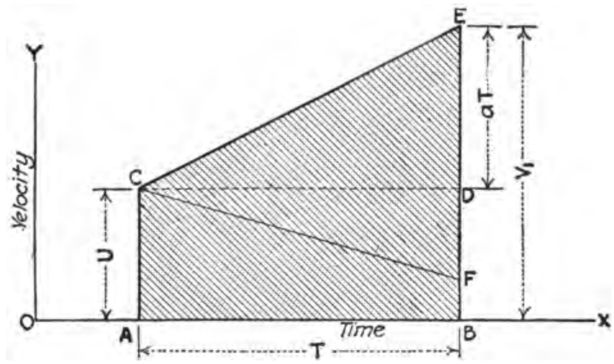

FIG. 65.

sec. and moves for T sec. with an acceleration of a ft. per sec. per sec. then the space passed over will be given by the area $ACEBA$, which may be divided into the two areas $ACDBA$ and $CDEC$.

$$\text{area } CDEC = aT \times \tfrac{1}{2}T = \tfrac{1}{2}aT^2$$

(The area of a triangle equals the product of one-half the base times the altitude); and the area $ACDBA$ equals UT. Hence

$$\text{area } ACEBA = UT + \tfrac{1}{2}aT^2$$

or the space passed over is the same as by equation (34).

Again, if the body is moving with a deceleration in addition to its initial velocity, then the space traversed will be given by the area $ACFBA$, where the line DF equals the loss in velocity

or aT (of course, in this case a is the deceleration in feet per second). But

area $ACFBA$ = area $ACDBA$ − area $CDFC$ = $UT - \frac{1}{2}aT^2$

which is the same as given by equation (38).

FALLING BODIES

Gravity attracts all bodies toward the earth with an acceleration which varies with the location of the body relative to the center of the earth. This acceleration is the same on all bodies, independent of their size, shape or weight. If a feather and a lead ball were put in a vessel from which all the air had been removed, it would be found that both objects would fall at the same rate. However, if the ball and the feather were dropped from the top of a building it is evident that the ball would reach the ground long before the feather, simply because the resistance of the air acts as a retarding force on the feather to a greater extent than it does on the ball. Further, if a body falls under the action of gravity alone, the velocity of the body will increase at a uniform rate. The acceleration, due to gravity, may be determined experimentally by the use of the Atwood machine, which will be described later. For all practical purposes, the value of this acceleration may be taken as 32.16 ft. per sec. per sec., and throughout the remaining lessons the letter g will be used to denote the *acceleration due to gravity.*

If a body be dropped from the top of a tower, the velocity will increase at the rate of g ft. per sec. for every second the body is falling, thus at the end of the first second the velocity will be 32.16 ft. per sec., at the end of the second second it will equal 2×32.16, or 64.32 ft. per sec., or, in general, *if an object starts from rest and falls under the action of gravity, the velocity at the end of T_1 sec. will be gT_1 ft. per sec.*

Let V = the final velocity in feet per second. Then,

$$V = gT_1 \tag{41}$$

The average velocity will be the sum of the initial and final velocities divided by 2 or

$$\frac{0 + gT_1}{2} \text{ or } \tfrac{1}{2}\, gT_1$$

and the distance the body has fallen in the given interval of time will equal

$$\tfrac{1}{2} gT_1 \times T_1 \text{ or } \tfrac{1}{2} gT_1^2$$

Let H equal the vertical distance the body has fallen. Then,

$$H = \tfrac{1}{2} gT_1^2 \qquad (42)$$

But, from equation (41)

$$T_1 = \frac{V}{g}$$

Substitute this value of T_1 in equation (42) and there results

$$H = \tfrac{1}{2}\, g \times \frac{V^2}{g^2} = \frac{V^2}{2g}$$

or

$$V^2 = 2gH \qquad (43)$$

Problem.—A wrench is let fall from the top of an elevator shaft which is 200 ft. deep. What is the velocity of the wrench when it reaches the bottom of the shaft? How long did it take the wrench to fall?

Solution.—To find the velocity substitute the value of H, which in this case is 200 ft., in equation (43) thus,

$$V^2 = 2gH = 2 \times 32.16 \times 200 = 12{,}864$$

or

$$V = 113.4 \text{ ft. per sec.}$$

The time of flight may be found from equation (41), where $V = 113.4$ and $g = 32.16$, hence

$$T_1 = \frac{V}{g} = \frac{113.4}{32.16} = 3.5 \text{ sec.}$$

Assume a body projected vertically upward with an initial velocity of U ft. per sec. Then neglecting the friction of the air, if it were not for gravity the body would continue to move up with a velocity of U ft. per sec. However, the instant the

body starts upward gravity exerts a deceleration of g ft. per sec. until the body is finally brought to rest at a certain height above the ground. The time that will elapse before the body comes to rest may be found by the use of equation (39) in which case $V_1 = 0$, $a = g$, $T = T_1$. Hence

$$0 = U - gT_1 \text{ or } U = gT_1$$

and,

$$T_1 = \frac{U}{g} \tag{44}$$

The instant the velocity becomes zero, the body will immediately start to fall with an acceleration of g ft. per sec. and at the end of a certain time will reach the ground with a velocity equal to that with which the body was projected upward.

Study Questions

81. Does the acceleration due to gravity increase or decrease with the distance of the body from the center of the earth?

82. An object dropped from the top of a steel stack, takes 4 sec. to reach the ground. Find the height of the stack.

83. A ball is thrown vertically upward with a velocity of 84 ft. per sec. When will the ball's velocity become zero?

84. In problem 82 how far was the object from the ground at the end of the second second?

85. A standpipe 50 ft. high is filled with water. With what velocity will the water leave a nozzle at the base of the standpipe? (Assume all the head converted into velocity.)

Answers

81. The acceleration due to gravity decreases the farther the body is from the center of the earth; that is, the acceleration of a body at the top of a mountain would be less than at the sea level.

82. Since the body is falling with an acceleration of 32.16 ft. per sec. per sec., the velocity at the end of 4 sec. will be

$$4 \times 32.16 = 128.6 \text{ ft. per sec.}$$

The average velocity during the entire 4 sec. was the sum of the initial and final velocities divided by 2, or

$$\frac{0 + 128.6}{2} = 64.3 \text{ ft. per sec.}$$

Therefore the body must have fallen

$$4 \times 64.3 = 257.2 \text{ ft.},$$

which distance equals the height of the stack.

83. The time that will elapse before the velocity of the body becomes zero may be found from equation (44), where $T_1 = \frac{U}{g}$. In this case $U = 84$ ft. per sec. and $g = 32.16$, hence

$$T_1 = \frac{84}{32.16} = 2.6 \text{ sec.}$$

84. The vertical distance through which an object falls, due to gravity, is given by equation (42), where $H = \frac{1}{2} g T_1^2$. If the time is 2 sec., the distance H will equal

$$\tfrac{1}{2} \times 32.16 \times 2^2 = 64.32 \text{ ft.}$$

In problem 82 the height of the stack was found to be 257.2 ft., hence at the end of the second second the object was

$$257.2 - 64.32 = 192.9 \text{ ft.}$$

from the ground. This shows that the object in the first two seconds (one-half the total time of flight) falls a distance of only one-fourth the height of the stack.

85. From equation (43) $V^2 = 2gH$. The velocity of the water emerging from the base of the standpipe will then equal the $\sqrt{2gH}$, or

$$\sqrt{2 \times 32.16 \times 50} = 56.7 \text{ ft. per sec.}$$

FALLING BODIES—*Continued*

The statement made at the close of the last lesson may be proved by substituting the proper values of a and S in equation (38). If a body is projected vertically upward, and then fal' back to its starting point, evidently the net space passed ov zero and hence the value of S in equation (38) become Therefore, when $S = O$ and $a = g$, there results

$$S = UT - \tfrac{1}{2} aT^2$$

or

$$O = UT - \tfrac{1}{2} gT^2$$

and

$$UT = \tfrac{1}{2} gT^2$$

hence,

$$T = \frac{2U}{g}$$

where T represents the whole time of f

In the example just solved, it was shown that the time of fall T_1 was equal to $\frac{U}{g}$. Therefore, the time of rise must equal the whole time of flight T minus the time of fall T_1, or

$$T - T_1 = \left(\frac{2U}{g} - \frac{U}{g}\right) = \frac{U}{g}$$

which is the same as the time of fall. Likewise the velocity at the end of the fall will be given by the equation $U = gT_1$, which is the same as the initial velocity of the body. Hence *a body projected vertically upward will require the same time to fall as it did to rise; and will reach the ground with the same velocity as that with which it was projected upward.*

The equations for falling bodies may then be summarized as follows:

$$V = gT_1 \tag{41}$$

$$H = \tfrac{1}{2} gT_1^2 \tag{42}$$

$$V^2 = 2gH \tag{43}$$

$$T_1 = \frac{U}{g} \tag{44}$$

where T_1 is the time of rise or fall in seconds; H is the vertical distance (in feet) the body has fallen; V is the velocity at the end of the fall, expressed in feet per second; and U is the velocity in feet per second with which the body was projected upward.

CHAPTER XII

LAWS OF MOTION

Thus far the motion of a body has been discussed without any reference being made to the forces producing the motion. For a clear understanding of the three laws of motion, as stated by Sir Isaac Newton, it will be necessary to define certain terms which are involved in these laws.

The *mass* of a body is the amount of material in the body. A cubic foot of iron and a cubic foot of wood occupy the same space, but experience teaches that the iron requires more force to put it in motion than does the wood, or if equal forces are applied to the two bodies the iron will move with a less acceleration than the wood.

The property of matter whereby force is required to change the state of rest or uniform motion of a body is called the *inertia* of the body. This property of matter accounts for the fact that it requires much more force to start or put a body in motion than it does to keep the body moving at a uniform rate. A fireman in the boiler room finds it much harder to start a coal truck in motion than he does to keep it moving across the boiler-room floor.

The *density* of a body is the mass of a unit volume of the body. The British unit of mass is called the Imperial pound, and consists of a lump of platinum deposited in the Exchequer office.

Let

$M =$ The mass of the body;
$V =$ The volume;

and $d =$ The density;

then,
$$M = V \times d \qquad (45)$$

or the mass of a body is equal to its density times its volume. The density of a body depends upon the relative closeness of the molecules of the body.

Momentum is the amount of motion in a body and is measured by the product of the mass times the velocity of the body. As defined earlier, force is that which changes or tends to change the state of rest or motion of a body. Hence the true measure of a force is the amount of motion destroyed or generated by the force in a given unit of time. Let m = the momentum of a body and V its velocity, then by definition,

$$m = M \times V \tag{46}$$

Law I.—Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it is compelled by imposed forces to change that state.

This law is proved by common observation. Thus the piston of an engine would continue in motion if it were not for external forces which are brought to bear upon it. A bullet shot from a gun would move in a straight line with a constant velocity if it were not for gravity exerting a downward pull upon it and the air offering a resistance to the motion of the bullet. This law shows the effect of the inertia of a body, for, to change the motion of a body, its inertia must be overcome and this can be done only by the application of forces external to the body itself.

Law II.—Change of motion is proportional to the impressed force and takes place in the line of action of the force.

As previously stated, the momentum is the quantity of motion in a body and therefore any force which produces a change of motion must be proportional to the momentum of the body. If the mass remains constant, the change in momentum will be dependent upon the change in velocity which, by definition, is the acceleration

Let

F = The effective force producing motion;

M = The mass of the body;

and

a = The acceleration.

Then the effective moving force (F) will equal the p the mass (M) times the acceleration (a) if the unit is taken as that force which will give to a unit mass in ond an acceleration of one foot per second per secor unit of force is called the *poundal*. Hence,

$$F = M \times a$$

The poundal is called the absolute unit of force be value is not dependent upon the force of gravity, whicl with the location of a body relative to the center of t

The *weight* of a mass of one pound is the force wi gravity draws the given mass toward the earth; hen equation (47), the weight of one pound is equivalent poundals of force, since gravity produces an accele 32.16 ft. per sec. per sec., or expressed as an equatio

$$W = M \times g$$

If the unit of force be taken as the weight of one will equal 32.16 poundals, or the poundal is equal ½ oz. avoirdupois. It is well to note that while the body remains constant, the weight of the body will pending upon the location of the body, for it has bee that the value of g varies. When the weight of one used as the unit of force, it is called the *gravitation force*, since its value depends upon the value of g.

Therefore in equation (47) the forces F may be exp pounds instead of poundals, if the value of M as given tion (48) be substituted, thus

$$F = \frac{W}{g} a$$

where

F = The effective moving force in pounds;
W = The whole mass moved in pounds;

and

a = The acceleration in feet per second per second.

Law III.—To every action there is an opposite and equal reaction. This law was discussed in Chapter I, to which the student is referred.

Study Questions

86. Due to the force of an explosion, a piece of boiler plate was projected vertically upward with a velocity of 100 ft. per sec. How far was the plate from the ground at the end of the second second?

87. What is the density of water; of coal? Explain the difference between density and specific gravity.

88. What force in poundals will produce an acceleration of 4 ft. per sec. per sec. on a mass of 100 lb. (neglect friction)?

89. An electric crane raises a flywheel weighing 10 tons with an acceleration of 1¼ ft. per sec. per sec. What is the tension in the hoisting cable?

90. If the wheel is brought to rest with a retardation of 2 ft. per sec. per sec., find the tension in the cable while the wheel is coming to rest.

Answers

86. If it were not for the force of gravity the distance the plate would move in T sec. would be UT ft.; but during the same time gravity acts and retards the motion of the body by an amount equal to $\frac{1}{2}gT^2$; therefore the total distance passed over by a body projected vertically upward may be found from the equation,

$$S = UT - \tfrac{1}{2}gT^2 \qquad (50)$$

where

U = The initial velocity of the body in feet per second;
T = The total time of flight in seconds.

If the value of S should work out negative it means that the body had reached its highest point and had returned to the ground before the completion of the assumed time. Furthermore, the above equation does not state whether the body is rising or falling, but this may be determined by ascertaining the time of ascent from the equation $T_1 = \frac{U}{g}$. In this problem

$$T_1 = \frac{100}{32} = 3+ \text{ sec.}$$

Hence at the end of the second second the plate is st height from the ground is found thus:

$$S = UT - \tfrac{1}{2}gT^2 = 100 \times 2 - \tfrac{1}{2} \times 32.16 \times 2^2$$

87. The density of water is the mass of a cubic foot about 39 deg. F. is 62.4 lb. per cu. ft.

The density of coal varies, depending upon the size An average value for anthracite coal is 96 lb. per cu. ft. heaps the density will average about 55 lb. per cu. ft.

The *specific gravity* of a material is its weight compar volume of water. Thus the density of wrought iron is 4 but its specific gravity is $\frac{480}{62.4} = 7+$.

88. From equation (47) $F = M \times a$. In this case th and the acceleration is 4 ft. per sec., hence the force equal to $100 \times 4 = 400$.

89. The weight of the flywheel is 10 tons, or 20,000 wheel is suspended from the crane hook the tension in th lb. (neglecting the weight of the block, cable, etc.), but is being put in motion the tension in the cable is incre equal to the force which is required to overcome the ine the desired acceleration. This force may be found fron where

$$F = \frac{W}{g} \times a$$

$$F = \frac{20{,}000}{32.16} \times 1.25 = 777.4 \text{ lb.}$$

Therefore, the total pull in the cable while the load i way is equal to

$$20{,}000 + 777.4 = 20{,}777.4 \text{ lb.}$$

90. When the flywheel is coming to rest its inertia will the tension in the cable. The force of inertia will equal

$$\frac{20{,}000}{32.16} \times 2 = 1244 \text{ lb.}$$

and the tension in the cable is equal to

$$20{,}000 - 1244 = 18{,}756 \text{ lb.}$$

CHAPTER XIII

WORK

A force is said to do *work* when it overcomes a resistance and causes an object to move in any given direction, or, as commonly expressed, *work is the overcoming of a resistance through a definite distance.* A horse moving a loaded van does work; the steam acting on the piston of an engine, by overcoming an external resistance, does work. An engineer may tug at the flywheel to move his engine off dead center, but no work will be done unless he has sufficient strength to turn the wheel. Hence the idea of work involves two fundamental notions; first, that of a resistance, and second, the overcoming of this resistance so that motion results. Moreover, the force exerted must be measured in the direction in which the resistance moves. There can be no work done on a body unless the body is put in motion.

The unit of work is the foot-pound. Thus, if a weight of one pound is raised through a vertical distance of one foot, the work done is one foot-pound; if the weight was 500 lb. and the distance 5 ft., the work done would equal 500×5 or 2500 ft.-lb. *The element of time does not enter into the idea of work;* thus the work done in raising the 500 lb. through the vertical distance of 5 ft. is 2500 ft.-lb., irrespective of whether it takes one minute or one hour to raise the weight.

A general equation for work may be expressed as follows:

Let W = the work done in foot-pounds,
F = the resistance overcome in pounds,
S = the space passed over by the resistance in feet,
then, since work = force $\times$ distance,

$$W = F \times S \tag{51}$$

A careful distinction must be made between the terms resistance overcome and the weight of the body moved.

Where bodies are raised in a vertical direction there the resistance overcome and the weight are equal to each other. Take, for example, a clam-shell bucket used for raising the coal from the barges to the bunkers of a power house. Here the resistance overcome by the hoisting engine is the weight of the bucket and the coal it contains. When this coal is placed in the distributing cars over the bunkers the force exerted by a cable to move a car is not equal to the weight of the car and its load, but to the force of friction between the car and the track. This force of friction is decidedly less than the weight of the car. When objects are moved along horizontal surfaces, the resistance overcome seldom equals the weight of the object moved. Hence in figuring the work done by a force in moving an object through a given distance the essential thing is to determine the resistance overcome and then the work may be readily determined.

Example.—A clam-shell bucket loaded with coal weighs three tons. How many foot-pounds of work will it require to raise the bucket a vertical distance of 200 ft.?

Here the resistance to be overcome is 6000 pounds and the distance is 200 ft., hence the work done is equal to

$$6000 \times 200 = 1{,}200{,}000 \text{ ft.-lb.}$$

Example.—Find the work done by a fireman in moving a coal truck weighing 1000 lb. across the floor of a boiler room 100 ft. wide, assuming the friction between the truck and floor as 50 lb. per ton.

In this problem the resistance overcome is not the weight of the car, but the friction between the car and the floor, which is equal to

$$\tfrac{1}{2} \times 50 \text{ or } 25 \text{ lb.}$$

since the car weighs one-half of a ton. The work done by the fireman is then equal to

$$25 \times 100 = 2500 \text{ ft.-lb.}$$

POWER

Power is the time rate of doing work or is the capacity of an agent for doing work at a uniform rate. The more work an agent can do in a unit of time the greater will be its power. The commonly accepted unit of power is the horsepower, which is the equivalent of 33,000 ft.-lb. of work being done every minute, or 550 ft.-lb. every second. This unit was named and adopted by Watt, as he figured that an ordinary dray horse was capable of doing 33,000 ft.-lb. of work per minute. Throughout this work the term horsepower will be expressed by the symbol *hp.*

Example.—In the case of the clam-shell bucket the work done was 1,200,000 ft.-lb. If the load had been raised in one minute, then the horsepower required would equal

$$\frac{1{,}200{,}000}{33{,}000} = 36.3+ \text{ hp.}$$

If the work had required 3 min. the hp. would have been only 12.1, so that the greater the time the less will be the power required to do a definite amount of work.

Let T = the time required to do a certain amount of work,
F = the resistance in pounds,
S = the space passed over by the resistance in feet,
hp. = the horsepower required, then

$$\text{hp.} = \frac{F \times S}{T \times 33{,}000} \tag{52}$$

EFFICIENCY

A machine is a device for the performance of useful work. Generally the machine will be supplied with power from some external source, and experience teaches that it is impossible to convert all the power supplied into useful work. There are numerous losses in the machine itself, such as the friction in the bearings, the loss in gears and belts, friction between sliding parts, and many others too numerous to mention.

The power supplied to the machine (commonly called the input) must equal the losses in the machine plus the power developed by the machine (commonly called the output.)

The *efficiency* of a machine is expressed in terms of the per cent. of the input converted into useful work or put in the form of an equation

$$\text{efficiency} = \frac{\text{output}}{\text{input}} \tag{53}$$

Example.—Assume the efficiency between the engine and the bucket (in previous example) to be 60 per cent. What horsepower engine would be required to raise the bucket?

The power required at the load was found to be 36.3 hp. As this (the output), represents only 60 per cent. of the input, it must be evident that the horsepower of the engine must be equal to the output divided by the efficiency, or

$$\frac{36.3}{0.60} = 60 \text{ hp.}$$

Study Questions

91. The average unbalanced steam pressure on the piston of an engine is 35 lb. per sq. in. If the diameter of the piston is 9 in. and the stroke of the engine is 12 in., find the work done by the steam in one revolution of

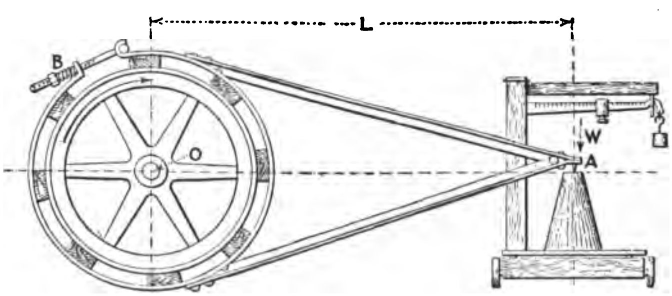

Fig. 66.

the engine. (Assume the engine to be double acting.)

92. In problem 91, if the engine makes 250 r.p.m., find the horsepower developed in the engine cylinder.

93. Fig. 66 shows a common form of prony brake used in the testing

of engines. During a test of the engine in problem 92 the length of the brake arm L was 5 ft., the net load on the brake was 120 lb. Find the horsepower developed at the brake.

94. From the results of problems 92 and 93 find the mechanical efficiency of the engine.

95. Explain what becomes of the power developed at the brake. Is it possible for the brake-horsepower to be greater than the power developed in the engine cylinder?

Answers

91. The area of the piston equals

$$0.7854 \times 9^2 = 63.62 \text{ sq. in.}$$

and the total pressure on the piston is equal to

$$63.62 \times 35 = 2227 \text{ lb.}$$

The length of the stroke is 12 in., or 1 ft., and, since the engine is double acting (that is, the engine takes steam on both sides of the piston), the piston will travel 2×1 or 2 ft. in one revolution of the engine; and the work done will equal

$$2227 \times 2 = 4454 \text{ ft.-lb.}$$

NOTE.—In an actual engine test, the mean effective steam pressure will generally be different for the head and the crank ends of the cylinder. On the crank end a correction must be made for the area of the piston rod, which will reduce the effective area of the piston.

92. The work done in one revolution was found to be 4454 ft.-lb., and hence the work done in 250 revolutions will equal

$$250 \times 4454 = 1{,}113{,}500 \text{ ft.-lb.}$$

Since this work was done in 1 min., the horsepower will be

$$\text{hp.} = \frac{1{,}113{,}500}{33{,}000} = 33.7+$$

93. The *prony brake* is a device for absorbing the power delivered by an engine, and consists of a series of wooden blocks fastened to a metal strap which passes around the rim of the wheel on which the brake is placed. By drawing together the ends of the strap, pressure may be put on the blocks, the value of which is determined by resting the end of the brake arm on platform scales. In figuring the horsepower developed, it is essential that the *net load* on the brake be used. This net load is found by subtracting the weight of the brake arm from the total load on the platform scales. For the sake of clearness in finding the brake

horsepower, assume the wheel as stationary and consider the weight of 120 lb. as rotating. The distance that the weight would travel in one revolution of the engine would equal the circumference of a circle whose radius is equal to the length of the brake arm. Hence the work done in one revolution will equal ($2 \times 3.1416 \times 5$) or 31.416 ft. times the net weight of 120 lb.,

$$31.416 \times 120 = 3770 \text{ ft.-lb. per revolution.}$$

(NOTE.—The circumference of a circle equals $2 \times 3.1416 \times$ the radius.)

The work done per minute will be

$$3770 \times 250 = 942{,}500 \text{ ft.-lb.}$$

and the developed horsepower will be

$$\frac{942{,}500}{33{,}000} = 28.6- \text{ hp.}$$

94. The output is the brake horsepower or 28.6, and the input is the indicated horsepower; that is, the power developed in the engine cylinder. (NOTE.—The term indicated horsepower is used because the average steam pressure is found from a diagram taken with an indicator.)

$$\text{efficiency} = \frac{\text{output}}{\text{input}} = \frac{28.6}{33.7} = 84.8+ \text{ per cent.}$$

95. The pressure between the blocks and the face of the pulley causes friction to be generated which is dissipated in the form of heat. To keep the pulley cool it is necessary to provide means of conducting away the heat generated. This is usually done by circulating water on the inside of a flanged pulley.

The brake or developed horsepower can never be greater than the power developed in the cylinder because friction will always be present in various parts of the engine.

GRAPHICAL REPRESENTATION OF WORK

Work has been defined as the product of a force times a distance. The area of any plane figure is the product of the base times the average height of the figure, and, as work involves but two terms, it may be represented graphically by the area of a figure whose base is proportional to the distance, and whose altitude is equal to the force. Thus, in a direct-acting steam pump, if the pressure on one side of the piston is

85 lb. per sq. in., and on the other side is 15 lb., then the average pressure will equal 70 lb. per sq. in. In Fig. 67 let the distance *BD* represent the stroke of the pump (assumed as 6 in.) and *AB* equal the unbalanced steam pressure. Then the area *ACDBA* will represent the work done by the steam on 1 sq. in. of the piston.

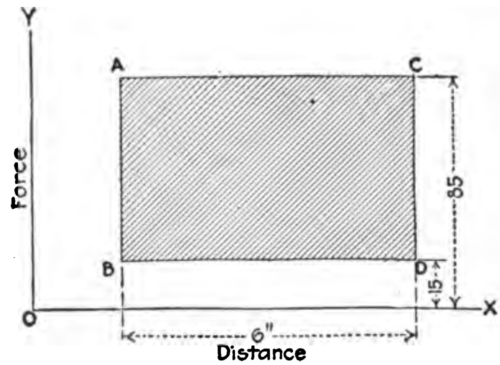

FIG. 67.

The area

$$ACDBA = AB \times BD = 70 \times 6 = 420 \text{ in.-lb.} = 35 \text{ ft.-lb.}$$

If the area of the piston is 20 sq. in., the total work done per stroke of the pump is

$$20 \times 35 = 700 \text{ ft.-lb.}$$

In like manner the area *ABCDA* of Fig. 68 represents the work done on 1 sq. in. of the piston of the steam engine provided the distance *EF* represents the stroke of the engine and the average height of the figure is proportional to the mean effective steam pressure in the engine cylinder. The average height of such a figure as shown may be found by dividing the diagram into 10 or more parts and finding the average height of each separate part, or it may be found more readily by the aid of an instrument called the planimeter.

As a general rule, the mean effective steam pressure will be found from the indicator diagram (Fig. 68) and the indicated horsepower of the engine may be found from the equation,

$$\text{i.hp.} = \frac{PLAN}{33{,}000} \tag{54}$$

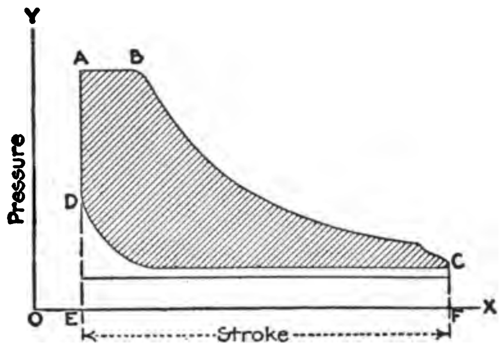

Fig. 68.

where

P_e = the mean effective steam pressure in pounds per square inch,
L = the length of the stroke in feet,
A = the area of the piston in square inches, and
N = the number of working strokes per minute.

In like manner, the developed or brake horsepower is expressed by the equation

$$\text{b.hp.} = \frac{2\pi LNW}{33{,}000} \tag{55}$$

where

L = the length of the brake arm in feet,
N = the number of revolutions per minute,
W = the net load on the brake in pounds, and
π = 3.1416.

CHAPTER XIV

ENERGY

Energy is the capacity of a body for doing work. Any moving body possesses a certain amount of energy which may be transformed into useful work as the body is brought to rest. Thus a stream of water issuing against the bucket of a water wheel has its energy converted into useful work by overcoming the resistance offered by the bucket. The sources of energy are almost unlimited; thus there is energy stored in coal and in all forms of liquid fuels; energy is stored in a coiled spring; the steam in a boiler possesses energy, and the human being has various forms of muscular energy.

Energy may be divided into two general classes,

(1) **Potential,** or energy of position, and

(2) **Kinetic,** or energy of motion.

The water in a reservoir has potential energy due to its location above a certain level. Energy was required to place the water at this level. The amount of potential energy in a body depends upon its weight and the distance of the body above a given reference line or plane. Thus, if an object weighing W pounds be located at a distance of H feet above a given datum plane, then the potential energy of the body will be expressed by the equation,

$$K = W \times H \tag{56}$$

The unit of energy is the same as that used for work, namely, the foot-pound.

An object possesses kinetic energy by virtue of its velocity. A bullet shot from a rifle is brought to rest by meeting with a body which offers a resistance to the motion of the bullet. The work done by the bullet is equal to the product of the

resistance overcome and the distance the bullet penetrated the object, and if frictional losses be neglected the work done by the bullet must equal the kinetic energy of the bullet.

In equation (49) it was shown that the force required to give a definite acceleration to a body weighing W pounds was equal to $\frac{W}{g} \times a$, and from equation (33) the acceleration was found to be $\frac{V^2}{2S}$ therefore, since

$$F = \frac{W}{g} \times a$$

and

$$a = \frac{V^2}{2S}$$

it follows that

$$F = \frac{W}{g} \times \frac{V^2}{2S}$$

or

$$F \times S = \frac{WV^2}{2g} \qquad (57)$$

But $F \times S$ is the product of a force times a distance, and hence represents the work done in bringing the body to rest. The term $\frac{WV^2}{2g}$ represents the energy stored in the body due to its velocity and is therefore the kinetic energy of the body.

Let

K_e = the kinetic energy in foot-pounds,
V = the velocity of the body in feet per second,
W = the weight of the body in pounds, then

$$K_e = \frac{WV^2}{2g} \qquad (58)$$

Study Questions

96. A pump delivers 1000 gal. of water per minute against a head of 120 ft. Find the horsepower required.

97. If the efficiency between the steam and water ends of the pump is 70 per cent., find the power required in the steam cylinder.

98. Find the horsepower of an engine that raises 20 tons of coal per

hour from a barge to a coal bunker located 150 ft. above the barge, assuming the efficiency between the engine and the load as 75 per cent.

99. A weight of 1000 lb. is used for crushing old castings. If the weight be raised to a distance of 95 ft. above the casting, find the potential energy of the weight.

100. Find the kinetic energy of the weight the instant it touches the casting, assuming that the weight was allowed to fall freely.

Answers

96. As 1 gal. of water weighs about $8\frac{1}{3}$ lb., the total mass of water raised is

$$1000 \times 8\tfrac{1}{3} = 8333\tfrac{1}{3} \text{ lb.}$$

Since this amount of water is lifted 120 ft. in one minute, the work done equals

$$8333\tfrac{1}{3} \times 120 = 1{,}000{,}000 \text{ ft.-lb.}$$

and the horsepower required to do this work is

$$\frac{1{,}000{,}000}{33{,}000} = 30.3 \text{ hp.}$$

97. The work done in the water end of the pump is equal to 30.3 hp. If the efficiency between the steam and water ends is 70 per cent., then the horsepower required in the steam end is

$$30.3 \div 0.70 = 43.3 \text{ h.p.}$$

98. Twenty tons of coal equals

$$2000 \times 20 = 40{,}000 \text{ lb.}$$

The vertical distance the coal is raised is 150 ft. From equation (52)

$$\text{Hp.} = \frac{F \times S}{T \times 33{,}000}$$

In this problem $F = 40{,}000$ lb., $S = 150$ ft. and $T = 60$ min., hence the horsepower required is

$$\frac{40{,}000 \times 150}{60 \times 33{,}000} = 3\tfrac{1}{3} \text{ hp. (theoretical)}$$

$$\text{Actual hp.} = \frac{3\frac{1}{3}}{0.75} = 4\tfrac{4}{9}$$

This answer holds true on the assumption that the engine is hoisting continuously for the entire hour. As an actual fact, such is not the case, as allowance must be made for the loading and lowering of the bucket, so that the actual time of hoisting is considerably less than one hour, and

hence the horsepower required will depend upon the rate at which the bucket is being raised in feet per minute.

99. Potential energy is weight times vertical distance above a given reference plane. Here the weight is 1000 lb. and the distance is 95 ft., hence the potential energy is

$$95 \times 1000 = 95{,}000 \text{ ft.-lb.}$$

100. The kinetic energy of the body is $\frac{W V^2}{2g}$. To determine this energy it is necessary to find the velocity V which the body has after it has fallen the vertical distance of 95 ft. This may be found from equation (43), where $V^2 = 2gH$, or

$$V = \sqrt{2gH} = \sqrt{2 \times 32.16 \times 95} = \sqrt{6102} = 78.2 \text{ ft. per sec.}$$

Substituting this value of V in the equation for kinetic energy there results

$$K_e = \frac{W V^2}{2g} = \frac{1000 \times 78.2 \times 78.2}{2 \times 32.16} = 95{,}000 \text{ ft.-lb.}$$

ENERGY—(*Continued*)

Problems 99 and 100 illustrate the fact that potential energy may be changed into kinetic energy by allowing the object to fall through the vertical distance above the given reference plane, or, as commonly expressed, by converting "head into velocity." This may be shown more clearly by comparison of the equations for potential and kinetic energy. Thus

$$K = WH$$

and

$$K_e = \frac{W V^2}{2g}$$

From equation (43)

$$V^2 = 2gH$$

or

$$H = \frac{V^2}{2g}$$

Substituting this value in the equation

$$K_e = \frac{W V^2}{2g},$$

there results

$$K_e = WH$$

which is the same as the expression for potential energy.

Moreover, *the total amount of energy in the body remains constant and is equal to the sum of the kinetic and potential energies.* In problem 100, had the body fallen only 47½ ft. its total energy would still have been 95,000 ft.-lb.; only 47,250 ft.-lb. would have been kinetic and 47,250 would have been potential energy.

Equation (57) shows that energy may be converted into useful work, and *vice versa;* but in no case is it possible to destroy or annihilate energy. This gives rise to the law commonly called the "*Conservation of Energy,*" which may be summed up in the following three statements:

(1) *The sum total of the energy in the universe remains constant.*

(2) *Energy may be converted from one form to another.*

(3) *Energy cannot be destroyed or annihilated.*

The first statement may not be evident at first sight. In a steam engine, due to improper lubrication, unlagged cylinders and poor materials, the efficiency may run very low and the engineer will say there is a loss of power. This statement is true as far as the performance of useful work is concerned, but there is no loss of energy. The steam by virtue of the energy it possesses does several things in the engine cylinder; first, it must overcome all frictional losses; second, it must heat up the cylinder, and, last of all, it must overcome the external resistance or load on the engine. Hence the energy in the steam is all accounted for and there is no loss, but simply a conversion of one form of energy into several other forms. Whatever energy may be lost in one form is sure to appear in a different form, so that the sum total of the energy of the universe remains constant.

That part of the energy of the steam which is not converted into useful work is called "*dissipated energy.*"

The second part of the law has already been referred to, that it is possible to change energy from one form to another. This change is universal and is taking place continuously. The human being eats food to provide him with muscular

energy which enables him to cultivate his fields and produce more food, which in turn is converted into muscular energy, and so the process continues. The ambition of the modern power plant engineer is to convert the heat energy resident in coal into electrical energy with as little loss as possible. Thus far his efforts have resulted in converting about 18 per cent. of the heat energy in the coal into the electrical form of energy. The introduction of the internal combustion engine is destined to raise this figure to a much higher value.

The third part of the law states that energy can never be lost. This is best illustrated by considering the action of the sun—the main source of the earth's energy. In the early ages the sun supplied the energy to plant life, which gradually developed into trees and into great forests. These in time perished and were buried deep in the earth, and after many years appeared in the form of coal, the energy of which may be liberated by combustion. Part of the coal's energy is transformed into work and the remainder is dissipated in various forms, but none of it is ever lost.

The sun by virtue of the energy it possesses raises the water from the level of the sea to the highest mountains, and as the water flows down from these elevations it becomes possible to convert its potential energy into kinetic energy, which may be utilized to operate waterwheels and turbines.

HEAT

Perhaps the most common form of energy is that known as *heat*, to which reference has already been made. For many years scientists were puzzled as to the form and nature of heat. Some considered it as a substance that had weight; but it remained for the simple experiment of rubbing two pieces of ice together to prove that heat is simply a form of molecular energy. As the molecules of the body move faster and faster, their kinetic energy becomes greater, and the sum total of all their energies constitutes what is known as heat.

The commonly accepted unit of heat is the *British thermal*

unit, which is the amount of heat required to raise the temperature of 1 lb. of water 1 deg. F., assuming the water to be at its greatest density, which is about 39 deg. F. Recent experiments have proved that the British thermal unit commonly indicated by the letters B.t.u. is equivalent to 780 ft.-lb. of work. A pound of coal contains about 13,000 B.t.u. If all the heat in the coal could be transformed into work there would result $13{,}000 \times 780 = 10{,}140{,}000$ ft.-lb., and if the heat was liberated in one minute the equivalent horsepower would be

$$\frac{10{,}140{,}000}{33{,}000} = 307.$$

Example.—Under certain conditions it takes 1050 B.t.u. to convert one pound of water into dry steam. If all the heat in the pound of coal could be transferred to the water, how many pounds would be evaporated for every pound of coal burned? The number of pounds of water evaporated per pound of coal will equal $\frac{13{,}000}{1050}$ or 12.4. In actual practice, only about 80 per cent. of this figure is realized. One horsepower is equivalent to 33,000 ft.-lb. per min., which expressed in B.t.u. equals $\frac{33{,}000}{780}$ or 42.3.

The figure 780 is called the mechanical equivalent of heat since it expresses the relation between the units of different forms of energy.

If one pound of coal were burned in one minute the horsepower equivalent to the heat energy in the fuel would be $\frac{13{,}000}{42.3}$ $= 307$, or if the coal was burned in one hour the horsepower equivalent would be $\frac{307}{60}$ or about 5. A boiler horsepower is equivalent to about 13 engine horsepower, so that the burning of one pound of coal per hour is capable of producing about $\frac{5}{13}$ boiler horsepower. Hence, to secure one boiler horsepower it is necessary to burn $\frac{13}{5}$ or $2\frac{3}{5}$ lb. of coal per hour. This of course is the theoretical value. Under average con-

ditions one boiler horsepower will be secured by the combustion of $3\frac{1}{2}$ to 4 lb. of coal per hour.

Study Questions

101. What three quantities determine the amount or quantity of heat in a given mass of material?

102. Explain the meaning of the term 1 deg. F.

103. Does the amount of heat required to change the temperature of water 1 deg. F. remain constant as the temperature of the water increases?

104. A certain coal averages about 12,000 B.t.u. to the pound. Find the horsepower equivalent to the combustion of 1000 lb. of coal per hour, assuming all the heat energy in the coal to be liberated during combustion.

What would be the equivalent boiler horsepower, assuming that 13 engine horsepower equal one boiler horsepower?

NOTE.—A boiler horsepower is the evaporation of $34\frac{1}{2}$ lb. of water per hour into dry steam at 212 deg. F. from a feed-water temperature of 212 deg. F. To evaporate 1 lb. under these conditions it requires 970.4 B.t.u. The heat equivalent of a boiler horsepower is equal to

$$34\frac{1}{2} \times 970.4 = 33{,}479 \text{ B.t.u. per hr.,}$$

or 558 B.t.u. per min. An engine horsepower is equal to 42.3 B.t.u. Therefore the boiler horsepower is equal to $\frac{558}{42.3}$ or 13 engine horsepower, approximately. As the efficiency between switchboard and coal pile is about 6 per cent., it is to be noted that one boiler horsepower under working conditions produces just about one engine horsepower.

105. In problem 104, if it takes 1050 B.t.u. to evaporate one pound of water, how many pounds would be evaporated under the given conditions, assuming that 80 per cent. of the heat in the fuel is transferred to the water in the boiler?

Answers

101. The three factors that determine the quantity of heat in a body are (a) the specific heat of the body. (NOTE.—*The specific heat of a body is the amount of heat required to raise one pound of the substance one degree Fahrenheit.*) (b) The weight of the body and (c) the range of temperature above a given reference point. As an example find the amount of heat in 100 lb. of water above 32 deg. F. if the temperature of the water is 62 deg. F. The specific heat of water is unity—the weight is 100 lb.—and the range of temperature above 32 deg. F. is $62 - 32 = 30$ deg.; hence the total heat in the water above 32 deg. F. is $1 \times 100 \times 30 = 3000$ B.t.u.

As a general rule the heat in a body is measured with reference to the temperature at which the body changes from a solid to a liquid, or a gas.

102. *Temperature expresses the degree or intensity of the heat in a body* and is measured by the rise of a column of mercury placed in a glass tube of fine uniform bore. On the Fahrenheit scale the freezing point of water is located at 32 deg. and the boiling point at 212 deg. Between the freezing and boiling points there are 180 divisions (212 − 32) on the scale. Hence, a change of 1 deg. F. means that the mercury rises a distance equal to $\frac{1}{180}$ of the distance between the freezing and the boiling points.

103. The quantity of heat required to change the temperature of 1 lb. of water 1 deg. F. decreases as the temperature of the water increases. This is due to the fact that the density of the water decreases with an increase of temperature.

104. The total heat liberated per hour by the complete combustion of 1000 lb. of the fuel is

$$12{,}000 \times 1000 = 12{,}000{,}000 \text{ B.t.u.}$$

and the heat liberated per minute

$$\frac{12{,}000{,}000}{60} = 200{,}000 \quad \text{B.t.u.}$$

As 1 hp. equals 42.3 B.t.u., the horsepower equivalent to the heat set free is

$$\frac{200{,}000}{42.3} = 4728 \text{ hp.}$$

The equivalent boiler horsepower is

$$\frac{4728}{13} = 363 \text{ b.hp.}$$

Under actual working conditions, only about 80 per cent. of the above figure could be secured, assuming that a water-tube boiler was used, so that the probable boiler horsepower would be about

$$0.8 \times 363 = 290 \text{ b.hp.}$$

105. From the conditions of the problem the heat absorbed by the water equals $0.80 \times 12{,}000{,}000$ or 9,600,000 B.t.u. per hour. To evaporate 1 lb. of water requires 1050 B.t.u., hence 9,600,000 B.t.u. will evaporate

$$\frac{9{,}600{,}000}{1050} = 9143 \text{ lb.}$$

Under working conditions, approximately 30 lb. of water must be evaporated to produce a boiler horsepower, hence the boiler horsepower is

$$\frac{9143}{30} = 304 \text{ b.hp.}$$

which agrees very closely with the answer given in problem 104.

CHAPTER XV

THE INCLINED PLANE

In Chapter VIII, the term "machine" was defined and several types were discussed. Some additional forms will now be discussed with relation to the work done by the machine.

Frequently, in construction work, heavy objects such as parts of machines, etc., must be lifted when, with the apparatus available, it is impracticable to raise the object directly. In such instances, the *inclined plane* is used. A common application of it is in the loading or unloading of heavy machinery from freight cars. A long incline is built in front of the car and the piece of machinery placed upon a skid which in turn rests upon the inclined plane. By the aid of the proper tackle the part can be drawn up to the car.

In the following discussion two general cases will be considered:

(1) The plane perfectly smooth—thus eliminating friction.
(2) The plane rough—thus including friction.

CASE I. SMOOTH PLANE—FRICTION NEGLECTED

Let AC, Fig. 69, represent the given plane inclined at an angle of a degrees with the horizontal and assume the body G to be acted upon by a force P parallel to the plane. The weight W may be resolved into two components—one parallel to the plane and the other at right angles to the plane. Since in this case friction is to be neglected, the only force tending to oppose the force P will be the component EF of the weight W down the plane. By construction, the angle EGF = the angle CAN = the angle a. The component $EF = W \times \sin a$, hence the force P must equal the component EF or $P = W \times \sin a$.

Let H = the height of the plane, and L = the length of the plane. Since $\sin a = \frac{CN}{AC} = \frac{H}{L}$ it follows that

$$P = W \times \sin a = \frac{W \times H}{L} \tag{59}$$

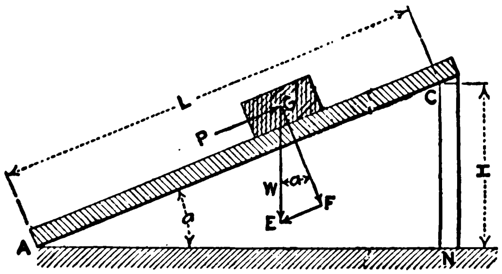

FIG. 69.

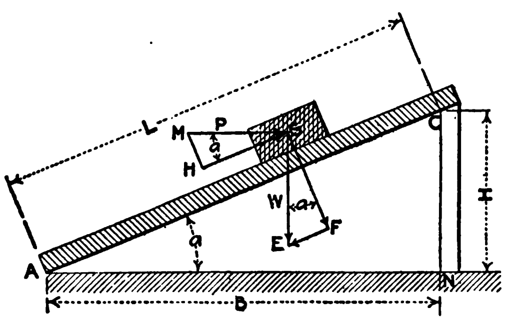

FIG. 70.

Example.—What force P will be required to move 1000 lb. on a plane which rises 1 ft. in 10. Here the

$$\sin a = \frac{H}{L} = 1/10$$

and the force

$$P = \frac{W \times H}{L} = \frac{1000 \times 1}{10} = 100 \text{ lb.}$$

This shows that a force of 100 lb. acting parallel to the plane is capable of moving 1000 lb. up the plane. Fig. 70 shows a

case where the force P is horizontal instead of being parallel to the plane. Here the only effective force tending to move the body up the plane will be the component of the force P parallel to the plane. By construction, it is readily seen that the angle HGM is equal to the angle CAN = the angle a. The component of the force P parallel to the plane = GH = $MG \cos a = P \cos a$. The force acting down the plane is, as before

$$W \times \sin a$$

For equilibrium to exist, the forces acting up the plane must equal the forces acting down the plane or

$$P \cos a = W \sin a$$

or,

$$P = W \frac{\sin a}{\cos a} = W \tan a \tag{60}$$

Let B = the length of the base of the plane. Then,

since $$\sin a = \frac{CN}{AC} = \frac{H}{L}$$

and $$\cos a = \frac{AN}{AC} = \frac{B}{L}$$

it follows that $$P = W \frac{\sin a}{\cos a} = W \frac{HL}{LB}$$

or

$$P = W \frac{H}{B} \tag{61}$$

Example.—What force P will be required to move 1000 lb. up a plane whose base angle is 5 deg. 50 min.? From equation (60)

$$P = W \tan a = 1000 \tan 5 \text{ deg. } 50 \text{ min.} = 1000 \times 0.102 = 102 \text{ lb.}$$

This demonstrates that the horizontal force required to move the body up the plane is greater than the force parallel to the plane.

CASE II. PLANE ROUGH—FRICTION INCLUDED

When a body is on the point of moving up the plane, both the force of gravity and the force of friction tend to oppose the motion, as both these forces are acting down the plane. On the other hand, if the body is on the point of moving down the plane the force of gravity will tend to help this motion, but the force of friction will act up the plane as *friction always acts opposite to the direction of motion.* Hence between certain limits the body will remain at rest on the plane.

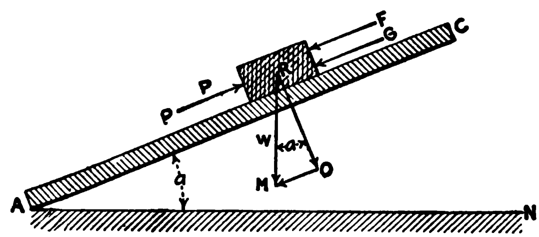

FIG. 71.

In Fig. 71, let the force P act parallel to and up the plane. Then the force of friction F down the plane will equal the coefficient of friction f times the normal reaction RO (see Chapter IX), and the force of gravity G, acting down the plane is equal to $W \sin a$. For equilibrium to exist $P = F + G$,

but $$F = f \times RO = f \times W \cos a,$$

therefore,

$$P = fW \cos a + W \sin a \tag{62}$$

The force P required to put the body in motion up the plane will be greater than that given by equation (62) by an amount equal to the force required to overcome the inertia of the body. When the force P has the value given in equation (62), the body will just be on the point of starting up the plane.

In Fig. 72, let the force P act up the plane and assume the body to be on the point of moving down the plane. In this case for equilibrium to exist, the force of gravity acting down

the plane must be balanced by the force of friction and the force P acting up the plane or

$$G = P + F, \text{ or } P = G - F$$

but $$F = f \times W \cos a$$

and $$G = W \sin a$$

therefore,

$$P = W \sin a - fW \cos a \tag{63}$$

For the body to remain at rest on the plane, the value of the force P must lie between the limits of $W\ (f \cos a + \sin a)$ and $W\ (\sin a - f \cos a)$.

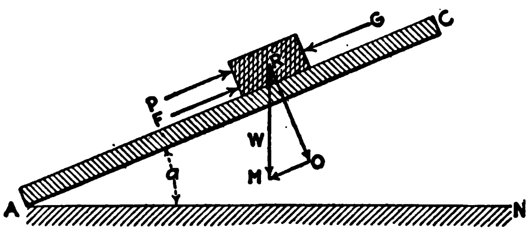

FIG. 72.

Study Questions

106. In Fig. 69, what force P will be required to hold a skid carrying a steam pump at rest on the plane AC? Neglect the force of friction. The plane rises 1 in 12 and the weight of the skid and pump is 1600 lb.

107. With the same data as given in 106, find the horizontal force P required to hold the skid on the plane.

108. If the coefficient of friction between the skid and the plane is 0.18, find the least force P which will prevent the skid from sliding down the plane.

109. Find the force P parallel to the plane which will just start motion of the skid up the plane. Assume the same data as in the previous problem.

110. Under what conditions will it be necessary for the force P to act down the plane to set the body in motion. See Fig. 72.

Answers

106. From equation (59) $P = \frac{W \times H}{L}$. In this problem $W = 1600$ lb.; $H = 1$ ft.; and $L = 12$ ft. so that

$$P = \frac{1600 \times 1}{12} = 133\tfrac{1}{3} \text{ lb.}$$

107. The horizontal force P necessary to hold the body at rest on the plane is $W\frac{\sin a}{\cos a} = W \tan a$. Since the plane rises 1 in 12 $\sin a = \frac{1}{12}$ or $a = 4$ deg. 47 min. Therefore the force

$$P = 1600 \times \tan 4 \text{ deg. } 47 \text{ min.} = 1600 \times 0.0836 = 133.8 \text{ lb.}$$

108. The force of friction F = the coefficient of friction f times the normal pressure, or

$$F = fW \cos a = 0.18 \times 1600 \times 0.9965 = 0.18 \times 1594 = 287 \text{ lb.}$$

The force of gravity tending to move the body down the plane is

$$W \sin a = 1600 \times 0.0833 = 133 \text{ lb.}$$

The force P necessary to prevent motion down the plane must equal the force of gravity minus the force of friction or,

$$P = G - F = W \sin a - fW \cos a = 133 - 287 = -154 \text{ lb.}$$

The fact that the value of P works out negative shows that under the given conditions the force of gravity is not sufficient to overcome the force of friction, so that to move the body down the plane an additional force P must be applied. (See answer to problem 110.)

109. For the body to be on the point of moving up the plane the force P must overcome both the force of gravity and the force of friction. From the previous problem the force of friction is 287 lb. and the force of gravity 133 lb. Therefore the force

$$P = F + G = 287 + 133 = 420 \text{ lb.}$$

Any value of P between -154 lb. and $+420$ lb. will hold the skid at rest on the plane.

110. The force P will act down the plane when the force of friction up the plane is greater than the force of gravity down the plane. This will occur when the tangent of the angle a is less than the coefficient of friction for it was shown in Chapter IX that when a body is just on the point of moving down a plane the coefficient of friction f is equal to the tangent of the angle made by the plane with the horizontal. (See solution of problem 108.)

INCLINED PLANE

As a final example under Case II let the force P, Fig. 73, act parallel to the base of the plane, and assume that the body is on the point of moving up the plane, under which condition both the force of gravity and the force of friction will act down the plane, thus opposing the upward force which is the component of P parallel to the plane.

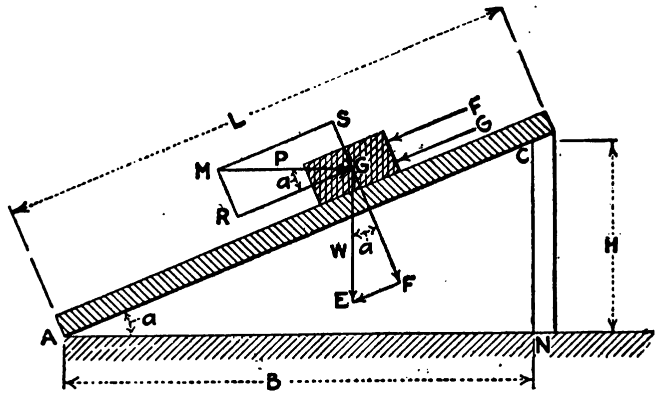

FIG. 73.

Resolve the horizontal force P into two components parallel to and at right angles to the plane by constructing the parallelogram $RMSGR$. The component RG, parallel to the plane will tend to move the body up the plane. The component SG will tend to increase the normal pressure between the weight and the plane, thus increasing the force of friction. Also resolve the weight W into two components parallel to and at right angles to the plane.

The force of gravity G, acting down the plane is equal to the component EF or $W \sin a$. The total normal pressure between the weight and the plane is the sum of the components SG and GF where

$$SG = MR = P \sin a$$

and

$$GF = W \cos a$$

From equation (22) the force of friction F, acting down the plane is equal to the total normal pressure N, times the coefficient of friction f, or,

$$F = f \times N$$

where

$$N = SG + GF = P \sin a + W \cos a$$

hence

$$F = f\,(P \sin a + W \cos a)$$

The effective force tending to move the body up the plane is the component of the force P parallel to the plane or RG which is equal to $P \cos a$. For equilibrium to exist, the forces F and G acting down the plane must equal the force RG, or $P \cos a$, acting up the plane, whence,

$$P \cos a = F + G$$

or

$$P \cos a = f\,(P \sin a + W \cos a) + W \sin a$$

To find the relation between P and W, solve the above equation by collecting all the terms involving P, thus,

$$P \cos a - fP \sin a = fW \cos a + W \sin a$$

now divide both sides of the equation by $\cos a$, then

$$\frac{P \cos a}{\cos a} - \frac{fP \sin a}{\cos a} = \frac{fW \cos a}{\cos a} + \frac{W \sin a}{\cos a},$$

or,

$$P - fP\frac{\sin a}{\cos a} = fW + W\frac{\sin a}{\cos a}$$

and

$$P(1 - f \tan a) = W(f + \tan a), \text{ since } \frac{\sin a}{\cos a} = \tan a,$$

therefore

$$P = W\left(\frac{f + \tan a}{1 - f \tan a}\right) \qquad (64)$$

When the weight W is on the point of moving down the plane then the force of friction will act up the plane or,

$$G = P \cos a + F$$

$$W \sin a = P \cos a + f(P \sin a + W \cos a)$$

and

$$P \cos a + fP \sin a = W \sin a - fW \cos a$$

hence, dividing through by $\cos a$,

$$P + fP \tan a = W \tan a - fW$$

or

$$P = W \frac{\tan a - f}{1 + f \tan a} \tag{65}$$

Any value of P between those given by equations (64) and (65) will hold the body at rest on the plane.

The efficiency of the inclined plane as a machine for the performance of useful work may be readily determined. In Fig. 73, when the weight has moved from A to C, the useful work performed is just the same as if the weight W had been raised in a vertical direction from the point N to the point C. Hence the net result of sliding the weight up the plane is the overcoming of a resistance of W lb. through a vertical distance of H ft. or the output of the machine is $W \times H$.

The work done on the body by the force P in moving the body up the plane is $P \cos a$ (resistance) times L (distance); but $L \cos a = B$ (the base of the plane), therefore the input $= P \times B$.

From equation (53) efficiency $= \dfrac{\text{output}}{\text{input}}$.

Let E = the efficiency of the inclined plane whose slope is a deg., then,

$$E = \frac{\text{output}}{\text{input}} = \frac{W \times H}{P \times B}$$

but $$\frac{H}{B} = \tan a$$

and $$P = W\left(\frac{f + \tan a}{1 - f\tan a}\right)$$

hence

$$E = \frac{W \tan a}{W\left(\dfrac{f + \tan a}{1 - f\tan a}\right)} = \frac{\tan a(1 - f\tan a)}{f + \tan a}$$

The above equation may be simplified by dividing numerator and denominator by tan a, thus,

$$E = \frac{1 - f\tan a}{\left(\dfrac{f}{\tan a}\right) + 1}.$$

or,

$$E = \frac{1 - f\tan a}{1 + f\cot a} \tag{66}$$

(NOTE.— $\dfrac{1}{\tan a} = \cot a.$)

Equation (66) is useful in determining the efficiency of a screw jack, a screw press or the efficiency of screw threads.

Turning a nut on a rod on which a thread has been cut is just the same as moving a body up an inclined plane by the aid of a horizontal force. The height of the plane NC, Fig. 73, corresponds to the pitch of the thread, the length L is the development of the thread, and the base B is the circumference of the bar on which the thread is cut. The inclination of the plane is the angle of the thread.

Example.—The angle of a given screw thread is 6 deg. The coefficient of friction between the nut and the thread is 0.15. Find the efficiency of the screw. In this case

$$\tan a = \tan 6 \text{ deg.} = 0.105$$

and

$$\cot 6 \text{ deg.} = 9.514$$

Substituting these values in equation (66), there results:

$$E = \frac{1 - f\tan a}{1 + f\cot a} = \frac{1 - (0.15 \times 0.105)}{1 + (0.15 \times 9.514)} = \frac{0.984}{2.427}$$

$$= 40.5 \text{ per cent.}$$

Study Questions

111. In the screw jack shown in Fig. 74, there are four threads per inch and the diameter of the screw is 2 in. Assuming the coefficient of friction as 0.14, find the efficiency of the jack.

112. A man exerts a pressure of 40 lb. at the point A, located 24 in. from the center of the jack. Find the maximum weight W that he can raise, neglecting the inertia of the weight.

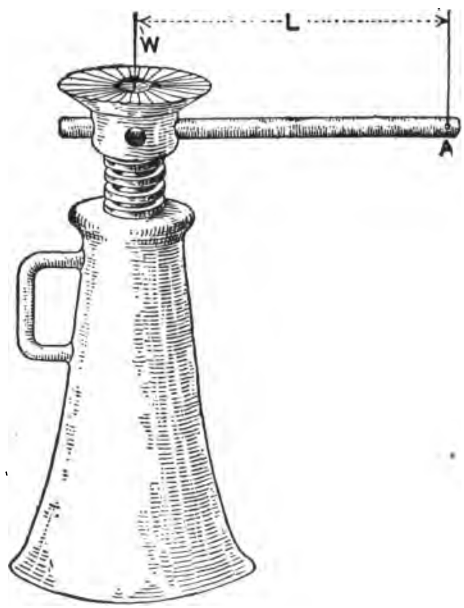

FIG. 74.

113. A loaded coal car weighing 2000 lb. starts down an incline whose slope is 1 in 18. If the frictional resistance is 20 lb. per ton, find the acceleration with which the car will move down the plane.

114. If the plane is 500 ft. long, find the velocity of the car when it reaches the foot of the incline.

115. How far will the car run on the level before coming to rest. (Figure the kinetic energy of the car when it reaches the foot of the incline and then find the distance from equation (57).)

Answers

111. Since there are four threads per inch, the pitch is ¼ in. Let Fig. 75 represent the development of the thread. The base of the plane is the circumference of the screw; the altitude is the pitch; and the hypotenuse AC is the length of the thread. The motion of the jack is equivalent to motion on the inclined plane AC.

By construction

$$\tan a = \frac{BC}{AB} = \frac{\frac{1}{4}}{6.28} = \frac{1}{25.12} = 0.04 \text{ (approximately)}$$

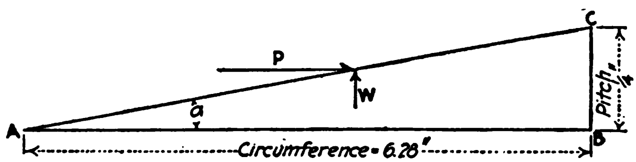

FIG. 75.

From a table of tangents the value of the angle a is found to be 2 deg. 20 min. From equation (66) the efficiency of the inclined plane is $E = \frac{1 - f \tan a}{1 + f \cot a}$. From the conditions of the problem the value of f is 0.14; the tan a = 0.04 and the cot a = 25. Substituting these values in equation (66), the efficiency of the jack is found thus:

$$E = \frac{1 - 0.14 \times 0.04}{1 + 0.14 \times 25} = \frac{0.9944}{1 + 3.5} = 0.22 \text{ or 22 per cent.}$$

112. In one revolution of the lever the point A will move a distance equal to 2 × 3.1416 × 2 or 12.56 ft. The force exerted by the man is 40 lb. and the work done per revolution of the jacks is 40 × 12.56, or 502 ft.-lb., which quantity represents the *input*.

Let W equal the weight raised. In one revolution of the jack this weight will move a vertical distance of ¼ in. or the pitch of the screw. Hence the output = $W \times \frac{1}{4} = W/4$ in.-lb., or $W/48$ ft.-lb. By equation 53 efficiency = $\frac{\text{output}}{\text{input}}$. In problem 111 the efficiency of the jack was found to be 22 per cent. Substituting the values of the efficiency, input, and the output in the above equation there results,

$$0.22 = \frac{W/48}{502} \text{ or } W = 0.22 \times 502 \times 48 = 5300 \text{ lb.}$$

113. The effective force moving the car down the plane is the force of gravity minus the force of friction. From the discussion of the in-

clined plane the force of gravity, G is $W \times \sin a$. In this problem $\sin a = 1/18$, hence

$$G = W \sin a = 2000 \times 1/18 = 111 \text{ lb.}$$

As the force of friction F is assumed to be 20 lb., the effective moving force is

$$(G - F) = (111 - 20) = 91 \text{ lb.}$$

The acceleration a may be found from equation (49), where $F = \frac{W}{g} \times a$, or

$$a = \frac{F \times g}{W} = \frac{91 \times 32.16}{2000} = 1.46 \text{ ft. per sec.}$$

114. Use the equation (32) $V^2 = 2aS$. Here $a = 1.46$ ft. per sec., and $S = 500$ ft.; therefore,

$$V^2 = 2 \times 1.46 \times 500 = 1460,$$

or

$$V = \sqrt{1460} = 38.2 \text{ ft. per sec.}$$

115. The car, including the load, weighs 2000 lb., and its velocity at the foot of the plane is 38.2 ft. per sec. The kinetic energy in the car, due to its velocity, is

$$\frac{WV^2}{2g}, \text{ or } \frac{2000 \times 38.2 \times 38.2}{2 \times 32.16} = 45{,}374 \text{ ft.-lb.}$$

This energy must be absorbed before the car will come to rest. When the car is running on the level the only resistance to be overcome is the force of friction which, in this case, is 20 lb. The work done in overcoming this resistance through a given distance of S ft. is $F \times S$ ft.-lb., which must equal the energy in the car, or $F \times S = \frac{W \times V^2}{2g}$, where $F = 20$ lb., and $\frac{WV^2}{2g} = 45{,}374$ ft.-lb. Hence, $S = \frac{WV^2}{2gF} = \frac{45{,}374}{20} = 2268$ ft.

CHAPTER XVI

PROJECTILES

In the previous lessons the motion of a body was limited to straight lines. There are cases where the path of the body may be a curve. If an object be projected vertically upward it will rise in a straight line to a certain height and then descend in the same straight line. If, however, the object be projected at an angle to the vertical the path described will be a curve, the form of which will be determined. In the following discussion the resistance of the air will be neglected, as an attempt to correct for it would require a knowledge of mathematics beyond the scope of this work. The acceleration due to gravity will be considered as uniform.

When an object is projected into the air, the angle that the direction in which it is projected makes with the horizontal is called the *angle of projection*. The path the object describes is called its *trajectory*. The distance from the point of projection to the point where the object again passes the plane of projection is called the *range*. The time that elapses before the object meets the plane of projection is called the *time* of *flight*.

As stated in Newton's second law of motion, a body would continue to move in a straight line forever if it were not for other external forces which are brought to bear on the body. The instant a bullet leaves a rifle gravity acts and tends to change the path of the bullet.

Projectiles may be divided into several classes, depending upon the angle of projection.

Class I. Bodies projected vertically upward. (This class has been discussed in Chapter XI.)

Class II. Bodies projected horizontally from a point at some elevation above a given reference plane.

Class III. Bodies projected at an angle to the horizontal.

In Fig. 76, assume an object to be projected horizontally from the point C with a velocity of V ft. per sec. Let H = the elevation, in feet, of the point C above the horizontal plane AB. It is desired to find at what point B the object will reach the ground and also the time of flight from the point C to the point B.

At the end of the first second the projectile would be at the point E, a distance of V ft. from the point C, if it were not for

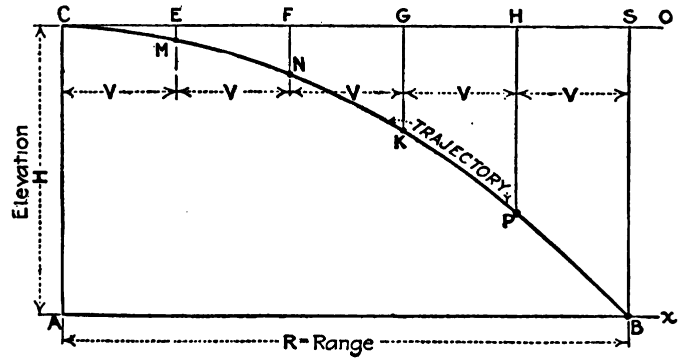

Fig. 76.

gravity, which in the same second causes the projectile to fall a distance of $\frac{1}{2}g$ ft. The net result is that at the end of the first second the body is not at the point E, but at a point M, 16 ft. below E. Likewise at the end of the second second the body would be at the point F, due to its initial velocity, but during these two seconds gravity causes it to fall a distance of

$$\frac{1}{2}gT^2 = \frac{1}{2} \times 32.16 \times 2^2 = 64 \text{ ft.}$$

so that the projectile is at the point N, which is located 64 ft. below F. At the end of the third second the projectile is at the point O, which is $4\frac{1}{2}g$, or 144 ft. below G. Hence to plot the true path of the projectile, the following method may be used:

Lay off the vertical distance H (Fig. 74) to a definite scale;

that is, 1 in. of height represents a fixed number of feet. Draw the horizontal reference line AX, and through the point C draw the line CO parallel to the line AX. On the line CO locate the points E, F, G, etc., so that the distance between any two consecutive points is equal to the initial velocity of the projectile in feet per second. Through the points E, F, G, etc., draw vertical lines, and on these lines lay off distances equal to the space through which a body would fall due to gravity in the given number of seconds. Thus at the end of the first second the body would have fallen a distance of 16 ft., and so the distance EM is made 16 ft. to scale. The distance FN is 64 ft., GK is 144, etc. A smooth curve drawn through the points C, M, N, K, B, will give the trajectory of the projectile.

The time of flight will be the same as if the projectile had been let fall from the point C and has nothing to do with the initial velocity of the projectile, but is entirely dependent upon the vertical distance AC. The simultaneous action of the velocity V and the action of gravity produce the same effect as though these forces acted independently of one another, that is the projectile may be considered as first moving from the point C to the point S with a velocity of V ft. per sec., and then falling, due to gravity, from the point S to the point B.

The time that it takes for the projectile to fall the vertical distance AC or H ft. may be found from equation (42) where $H = \frac{1}{2} gT^2$; and when the time is known the horizontal distance AB or the range is found from the equation $S = V \times T_1$. Therefore, let T_1 = the whole time of flight; H = the vertical distance from the plane AB to the point of projection; V = the velocity of projection in feet per second; and R = the horizontal range or the distance AB.

Then
$$T_1 = \sqrt{\frac{2H}{g}} \tag{67}$$

and
$$R = V \times T_1 \tag{68}$$

or
$$R = V \sqrt{\frac{2H}{g}} \tag{69}$$

The above formulas apply only to problems coming under Class II.

Study Questions

116. A horizontal belt conveyor is located 100 ft. above the ground. Coal leaves the conveyor with a horizontal velocity of 20 ft. per sec. How long after the coal leaves the conveyor will it strike the ground?

117. Find the horizontal distance from the conveyor to the point where the coal strikes the ground.

118. Find the magnitude and direction of the velocity of the coal when it strikes the ground.

119. In Fig. 14, the governor ball B is 15 in. from the center of the spindle when revolving at 100 r.p.m., and is 15 ft. above the floor line. If the ball were to suddenly break loose from the spindle, how far would it travel before striking the ground?

120. What would be the time of flight?

Answers

116. The time of flight, which is independent of the velocity of projection, may be found from equation (67), where $T_1 = \sqrt{\frac{2H}{g}}$. As the conveyor is 100 ft. above the ground the value of H is 100 ft., hence

$$T_1 = \sqrt{\frac{2 \times 100}{32.16}} = \sqrt{6.22} = 2.5 \text{ sec.}$$

117. While the coal is falling it is also moving horizontally at the rate of 20 ft. per sec. From problem 116 the coal strikes the ground 2.5 sec. after leaving the conveyor; therefore, the point where the coal lands is at a horizontal distance of

$$2.5 \times 20 = 50 \text{ ft. from the conveyor.}$$

118. At the instant it strikes the ground, the coal has two velocities—one of 20 ft. per sec. horizontally, represented by the line AB, Fig. 77, and a vertical one, due to gravity, represented by the line AD. The value of the velocity, due to gravity, is given by equation (41) where $V = gT_1$. In this case the time of flight is 2.5 sec., hence the velocity $AD = 32.16 \times 2.5 = 80.4$ ft. per sec. The resultant velocity is given by the diagonal AC of the parallelogram $ABCDA$.

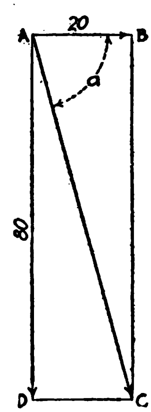

FIG. 77.

B $$\overline{AC}^2 = \overline{AD}^2 + \overline{DC}^2 = \overline{80}^2 + \overline{20}^2 = 6800,$$

or $$AC = \sqrt{6800} = 82.5 \text{ ft. per sec.}$$

The resultant AC makes an angle BAC (see Fig. 77) with the horizontal. But $\sin BAC = \sin a = \dfrac{BC}{AC} = \dfrac{80}{82.5} = 0.97$. From a table of sines the angle a is found to be 76 deg.

119. The linear velocity of the ball is equal to $2 \times 3.1416 \times R \times N$, where R = the distance from the center of the spindle to the center of the ball, and N = the r.p.m., or

$$V = 2 \times 3.1416 \times {}^{15}\!/_{12} \times 100 = 785 \text{ ft. per min., or 13 ft. per sec.}$$

If the ball were to break loose from the arm, it would move horizontally with a velocity of 13 ft. per sec., and at the same time gravity would pull it toward the ground. The time that would elapse before the ball strikes the ground is dependent upon the height of the ball from the ground, which is 15 ft. From equation (42),

$$H = \tfrac{1}{2} g T_1^2, \text{ or } T_1 = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 15}{32.16}} = 1 \text{ sec. (approx.).}$$

During this same time the ball would travel 13 ft., that is, the ball would strike the ground 13 ft. from the governor.

120. From problem 119, the time of flight is found to be 1 sec.

CLASS III. BODIES PROJECTED AT AN ANGLE TO THE HORIZONTAL

In Fig. 78, assume an object to be projected from the point A with a velocity of V ft. per sec., at an inclination of a deg. with the horizontal, and let the line AB represent this velocity. Through the point A draw the horizontal and vertical reference lines AK and AN and on the diagonal AB construct the parallelogram $ANBMA$. The line AM will then represent the horizontal component of the velocity V, and will give the rate at which the projectile is moving horizontally. Likewise the line AN will represent the vertical component of the velocity V and gives the rate at which the projectile is moving vertically. The only force opposing the component V_1 is the resistance of the air, which, in this discussion, is assumed as zero, hence as long as the projectile is above the ground it will have a uniform horizontal displacement of V_1 ft. per sec.

The component V_2 is being reduced at a uniform rate by the action of gravity until finally it becomes zero, at which instant the projectile has reached its maximum elevation above the ground as represented by the point S, in Fig. 78. From this point on, the action of gravity is greater than the vertical velocity, with the result that the projectile is gradually drawn toward the earth and finally reaches it at the point K.

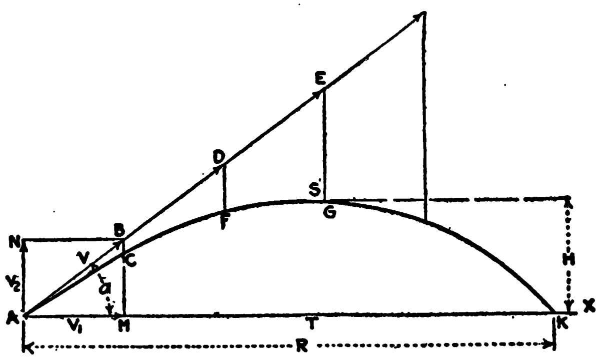

FIG. 78.

The distance $ST = H$ gives the maximum height the body will rise and the distance $AK = R$ represents the range of the projectile.

At the end of the first second of flight the projectile would be at the point B, distant V ft. from the point A, but during this second gravity acts and pulls the object toward the earth a distance of $\frac{1}{2}\ gT_1^2$ ft., or 16 ft., so that the true position of the projectile is at C, 16 ft. below B. Likewise at the end of the second second the projectile is at F, located $\frac{1}{2}\ gT_1^2$, or 64 ft. below D.

The path of the projectile may be plotted in a manner similar to that explained under Case II. Thus, through the point of projection A draw the line AK, and also the line AE at an angle of a deg. to AK, to represent the direction of projection. On this line locate the points A, B, D, E, etc., distant V, $2V$, $3V$, etc., from A. Through the points B, D, E, etc.,

draw the vertical lines BC, DF, EG, etc., and on these lay off to scale the distances through which a body would fall due to gravity in the given number of seconds. Thus at the end of the first second, the prejectile would be at C, at the end of the second second at F, etc. A smooth curve drawn through these points will give the path or the trajectory of the body.

In Chapter XI, it was shown that if a body be projected vertically upward with a given velocity that the total time of flight would be $T = \frac{2U}{g}$ sec., where U is the vertical velocity of projection. In this case the vertical velocity is the vertical component of the velocity V, or $V \sin a$, hence when a body is projected at an angle of a deg. to the horizontal with a velocity of V ft. per sec., the total time of flight will equal $\frac{2 \times V \sin a}{g}$, or,

$$T_1 = \frac{2 V \sin a}{g} \tag{70}$$

The range, or horizontal displacement of the projectile, will equal the horizontal component V_1, or $V \cos a$, times the whole time of flight, or,

$$R = V \cos a \times T_1 \tag{71}$$

Now substituting the value of T_1 from equation (70), in equation (71), there results,

$$R = V \cos a \frac{2 V \sin a}{g} = \frac{2 V^2 \sin a \cos a}{g}$$

but from the Trigonometry, $2 \sin a \cos a = \sin 2a$, therefore,

$$R = \frac{V^2 \sin 2a}{g} \tag{72}$$

When the angle $a = 45$ deg., the angle $2a = 90$ deg., and $\sin 2a = \sin 90$ deg. $= 1$, and hence the horizontal range $R = \frac{V^2 \sin 90 \text{ deg.}}{g} = \frac{V^2}{g}$.

This demonstrates that a projectile will have its maximum range when the angle of projection is 45 deg.

The maximum height H that the projectile will rise above the horizontal is dependent upon the vertical component V_2 of the velocity V. Thus the value of H may be found from equation (43) by substituting for V the velocity $V_2 = V \sin a$, thus,

$$H = \frac{V^2 \sin^2 a}{2g} \tag{73}$$

The velocity with which the projectile is moving vertically at any given instant during the flight may be found by considering that there are two velocities acting—one of $V \sin a$ upward, due to the initial velocity of projection, and one of gT ft. downward, due to gravity. Let U = the vertical velocity at the end of T sec., then,

$$U = V \sin a - gT \tag{74}$$

where T = the time that has elapsed from the instant of projection.

If the value of U works out negative, it indicates that the projectile has reached its maximum elevation and has started downward.

There is a special case coming under Class III, namely, when a body is projected from a point at a given elevation from the horizontal, as, for example, the firing of a projectile from a cliff. The time of flight in such a problem may be determined as follows:

Let H = the elevation of the point of projection. Then the total net vertical displacement of the projectile will be $-H$ ft., as the projectile must rise to its highest point and then fall H ft. below the point of projection. The vertical velocity is $V \sin a$. From equation (38), $S = UT - \frac{1}{2} gT^2$, and as $S = -H$, and $U = V \sin a$, it follows that,

$$-H = V \sin a \times T - \tfrac{1}{2} gT^2 \tag{75}$$

where T is the total time of flight.

Study Questions

121. A 12-in. gun discharges a projectile with a velocity of 1200 ft. per sec. If the gun is inclined to the horizontal at an angle of 45 deg., find the time of flight.

122. At what distance from the gun will the projectile strike the ground?

123. Find the maximum distance that the projectile rises above the ground.

124. If the gun is on a cliff 500 ft. above the sea, where would the projectile strike the water if fired under the conditions stated in problem 121?

125. What is the total time of flight?

Answers

121. The time of flight is found from equation (70), where $T_1 = \frac{2V \sin a}{g}$. In this case the angle a = 45 deg. and $\sin a = 0.707$; also V = 1200 ft. per sec., hence

$$T_1 = \frac{2 \times 1200 \times 0.707}{32.16} = 52.8 \text{ sec.}$$

122. The projectile will strike the ground at a distance from the gun equal to the horizontal range R, which is determined from equation (71), where $R = V \cos a \times T_1$; hence

$$R = 1200 \times 0.707 \times 52.8 = 44{,}795 \text{ ft., or } 8.48 \text{ miles.}$$

123. From equation (73), $H = \frac{V^2 \sin^2 a}{2g}$. In this problem V = 1200 ft. per sec.; $\sin a = 0.707$, hence

$$H = \frac{1200 \times 1200 \times 0.707 \times 0.707}{2 \times 32.16} = 11{,}190 \text{ ft. or } 2.12 \text{ miles.}$$

124. The range is easiest determined by first solving problem 125, to ascertain the time of flight. From equation (75), $-H = V \sin a \times T - \frac{1}{2} g \times T^2$, where H is the elevation of the point of projection and in this case is equal to 500 ft. Substituting the given values in equation (75) there results, $-500 = 1200 \times 0.707 \times T - 16.08\, T^2$ or, $T^2 - 52.76\, T = 31.09$, which is a quadratic equation and may be solved by adding 695.9 to both sides of the equation, so as to make the left side a perfect square, thus,

$$T^2 - 52.76\, T + 695.9 = 31.09 + 695.9 = 727$$

and taking the square root of both sides, there results,

$$T - 26.38 = 26.96$$

or,

$$T = 26.96 + 26.38 = 53.3 \text{ sec.}$$

The range is dependent upon the horizontal component of the velocity and the time of flight, or, as given by equation (71), $R = V \cos a \times T_1$; where $V = 1200$ ft. per sec., and $T_1 = 53.3$ sec., therefore

$$R = 1200 \times 0.707 \times 53.3 = 45{,}220 \text{ ft., or } 8.58 \text{ miles.}$$

125. In problem 124 the total time of flight was found to be 53.3 sec., which, it will be noted, is just about ½ sec. longer than the time found in problem 121.

CHAPTER XVII

ROTARY MOTION

In equation (47), it was shown that the force required to produce an acceleration of a ft. per sec. per sec. on a mass of M lb. was equal to $M \times a$. As the mass becomes greater the force required to produce a given acceleration also becomes greater; that is, the force is directly proportional to the mass, *assuming that the mass is moving in a straight line.*

In the case of rotating bodies, not only must the mass be considered, but also its distance from the center of rotation. Thus a small force acting at a great distance from the center of rotation may produce the same "torque" or turning moment as a large force acting at a small distance. The resistance to rotation of a given body, then, depends both on the mass of the body and the distribution of the mass relative to the center of rotation. It is evident from experience that it is much easier to set in motion a flywheel whose mass is concentrated in its hub than it is to set in motion one whose mass is concentrated in its rim. Moreover, it must be apparent that the linear acceleration of a given mass is also dependent upon its distance from the center of rotation. Thus, if two wheels of the same mass, but of different diameters, are to be brought up to speed in the same time, the linear accelerations of their rims must be different in order to produce the same angular acceleration.

In equation (26) it was shown that the angular velocity of a body, expressed in radians, was equal to the linear velocity divided by the radius. In like manner it may be shown that the angular acceleration in radians is equal to the linear acceleration in feet per second per second, divided by the radius in feet. Thus, let

a = The linear acceleration in feet per second per second.
a_1 = The angular acceleration in radians per second per second.
R = The radius of rotation in feet, then,

$$a_1 = \frac{a}{R} \text{ or, } a = a_1 \times R$$

As a general rule, the acceleration of rotating bodies is expressed in revolutions per second per second, and as there are 2π radians in the circumference of a circle it follows that $a = 2\pi AR$, where A = the increase in revolutions per second per second. In the case of a flywheel, each separate particle will be located at a fixed distance from the center of rotation and, as there is an infinite number of particles, there will be an infinite number of radii and each particle will have a different linear velocity. For this reason the mass of the wheel is assumed concentrated at a certain point, so that the resistance which the wheel would offer to a change of angular velocity will remain the same. The distance from this point to the center of rotation is called the *radius of gyration* of the wheel. The value of the radius of gyration of various sections may be found by reference to any engineers' handbook. In the case of flywheels with heavy rims the hub and arms are neglected and the weight of the rim only is considered. In such cases, it is customary to consider the radius of gyration as the distance from the center of the wheel to the middle of the rim.

In Fig. 79, assume a heavy wheel A to be mounted on the shaft C, to which is keyed the pulley B. A force of P lb. is applied at the rim of the pulley. It is desired to find the value of this force in terms of the weight of the wheel in order to give a definite angular acceleration to the wheel. First assume that the mass M of the wheel is concentrated at the center of the rim at a distance of R ft. from the center of the shaft. It must be evident that the inertia of the rim will act in a direction opposite to that in which the wheel is to move.

Let F = the force of inertia which, from equation (47), is equal to the mass of the rim times the linear acceleration, or $F = M \times a$. Now from the conditions of equilibrium, the algebraic sum of the moments about the point O must equal zero. Thus, $F \times R = P \times d$, and as $F = M \times a$, it follows

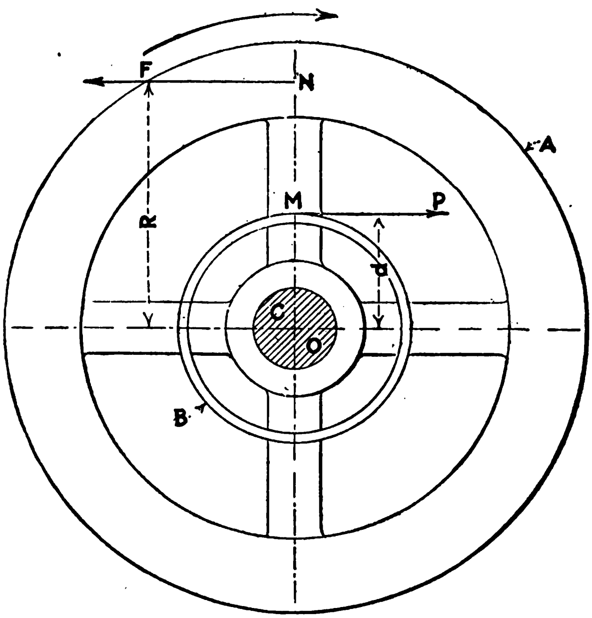

FIG. 79.

that, $M \times a \times R = P \times d$. Also as previously proven, the linear acceleration a is equal to $2\pi A \times R$; therefore, substituting this value in the above equation, there results,

$$M \times 2\pi \times A \times R \times R = P \times d, \text{ or,}$$

$$2\pi \times MR^2 \times A = P \times d \qquad (76)$$

Expressed in gravitational units, the equation becomes,

$$2\pi \times \frac{W}{g} \times R^2 \times A = P \times d \qquad (77)$$

where,

W = The weight of the rim in pounds.
R = The radius of gyration in feet.
A = The angular acceleration in revolutions per second per second.
P = The force required in pounds.
d = The moment arm in feet of the force P.
g = Acceleration due to gravity = 32.16.

The quantity MR^2 is called the moment of inertia of the body. To determine this quantity, it is necessary to consider the body as being made up of an infinite number of small particles, and to multiply each of these particles by the square of its distance from the center of rotation, and then add all these small quantities together, which sum gives the moment of inertia.

The moment of inertia of bodies of uniform density may be determined by the aid of calculus, which is beyond the scope of this work. However, these values may be found by reference to handbooks.

In the design of flywheels, it is desired to have as large an inertia effect as possible with a given weight. For this reason the moment of inertia is made large by placing most of the metal in the rim and making the diameter of the wheel relatively large. The moment of inertia of the hub and the arms is comparatively small, and, therefore, in the following problems only the inertia effect of the rim will be considered.

Study Questions

126. The point N on the wheel A, Fig. 79, is 5 ft. from the center of the shaft. The wheel starts from rest and at the end of 2 min. is running at 75 r.p.m. Find the linear acceleration of the point N in feet per second per second.

127. (a) Find the angular acceleration of the point N in revolutions per second per second. (b) Do all points in the wheel have the same angular acceleration?

128. The outside diameter of a cast-iron flywheel is 12 ft. The rim

is 8 in. thick and the face of the wheel is 10 in. Find the weight of the rim and its radius of gyration.

129. What force P, acting at a distance of 2 ft. from the center of the wheel, will be required to bring the wheel up to a speed of 90 r.p.m. in 1½ min. Neglect the weight of the hub and the arms.

130. Assume the wheel running at 120 r.p.m. What braking force P, acting at a distance of 18 in. from the center of the shaft, will be required to reduce the speed to 60 rev. in 1 min.?

Answers

126. The linear velocity of the point N when the wheel is running at 75 r.p.m. is equal to the circumference of a circle whose radius is 5 ft., times the r.p.m. Thus, the velocity of the point $N = 2 \times 3.1416 \times 5 \times 75 = 2356$ ft. per min., or 39.3 ft. per sec. As this velocity was acquired in 2 min., or 120 sec., the gain in velocity in feet per second per second, or the acceleration, is

$$\frac{39.3}{120} = 0.327 \text{ ft.}$$

127. (a) In Chapter XVII, it was shown that the gain in velocity in revolutions per second per second was equal to the linear acceleration a, divided by the radius times 2π. Thus,

$$A = \frac{a}{2 \times \pi \times R} = \frac{0.327}{2 \times 3.1416 \times 5} = 0.0104$$

which gives the acceleration in revolutions per second per second.

This acceleration might be found in another way. Thus, the revolutions per second $= 75/60 = 1.25$, and as it required 120 sec. to attain this speed, evidently the gain in revolutions per second per second must be $\frac{1.25}{120} = 0.0104$.

(b) All points in the wheel have the same angular acceleration, as they all pass through the same number of radians per second.

128. The weight of the wheel is the same as that of a bar 8×10 in. in cross-section, and of a length equal to the circumference of a circle whose diameter is equal to the mean or average diameter of the rim. As the outside of the wheel is 12 ft. in diameter, and the rim is 8 in. thick, it follows that the mean diameter of the rim is 11 ft. 4 in. The problem now resolves itself into finding the weight of a bar of cast iron 8×10 in. in cross-section and 427¼ in. long (the circumference of the circle). The cubical contents of the bar equal $8 \times 10 \times 427.25 = 34{,}180$ cu. in., or $\frac{34{,}180}{1728} = 19.78$ cu. ft. The weight of a cubic foot of cast iron is 450 lb., hence the total weight of the rim is $19.78 \times 450 = 8900$ lb.

The radius of gyration is the distance from the center of the shaft to the center of the rim, or 5 ft. 8 in.

129. A speed of 90 r.p.m. = 1.5 rev. per sec. As this speed was attained in $1\frac{1}{2}$ min., or 90 sec., the gain in speed was $\frac{1.5}{90} = 0.0166$ rev. per sec. per sec. In equation (77), it was shown that $P \times d = 2 \times \pi \times \frac{W}{g} \times R^2 \times A$, or, $P = \frac{2\pi W R^2 A}{gd}$. Now in this problem the weight W is 8900 lb.; the radius of gyration R is 5 ft. 8 in., or $\frac{17}{3}$ ft.; the angular acceleration A is 0.0166 rev. per sec. per sec.; $g = 32.16$, and the moment arm d is 2 ft. Hence, substituting these values in the above equations, there results,

$$P = \frac{2 \times 3.1416 \times 8900 \times 17 \times 17 \times 0.0166}{32.16 \times 2 \times 3 \times 3} = 463.4 \text{ lb.}$$

130. The reduction in speed is $(120 - 60) = 60$ r.p.m., or 1 rev. per sec. As this reduction takes place in 1 min., or 60 sec., the deceleration in revolutions per second per second $= \frac{1}{60}$. The braking force P required may be found from the same equation as used in the previous problem for finding the accelerating force; thus,

$$P = \frac{2\pi \times W \times R^2 \times A}{g \times d}$$

where $W = 8900$ lb.; $R = \frac{17}{3}$ ft.; $A = \frac{1}{60}$ and $d = 1.5$ ft., therefore,

$$P = \frac{2 \times 3.1416 \times 8900 \times 17 \times 17 \times 1}{32.16 \times 1.5 \times 3 \times 3 \times 60} = 620.4 \text{ lb.}$$

CHAPTER XVIII

CENTRIFUGAL FORCE

Newton's first law of motion states that all bodies continue to move in a straight line except in so far as compelled by external forces to change that path. In order that a body may move in a circle, it becomes necessary to have some constraining force applied. If a flywheel breaks, it is a matter of com-

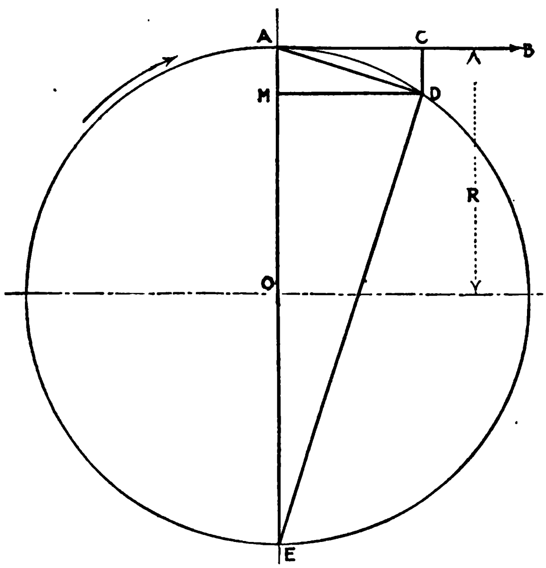

Fig. 80.

mon experience that the parts tend to move in straight lines because the motion is no longer constrained. What force, then, is necessary to keep an object moving in a circle?

In Fig. 80, let the point A be rotating with a uniform linear velocity V about the point O. Now if the point A is to be kept

moving in a circle, it must be evident that the link OA must exert a force sufficient to hold the point A in its path. This force is called the *centripetal force,* since it tends to draw the point A toward the center O with a definite acceleration. Likewise, the force tending to draw the body A away from the center is called the *centrifugal force.* From the conditions of equilibrium, the centripetal and the centrifugal forces must be equal and opposite to each other. If the body A were free, it would move along the line AB, and at the end of a given time would be at some point C, but instead of being at C, due to the centripetal force, the point A rotates about the point O and reaches some point D on the circumference of the circle. Connect the points A and D with a straight line. Now if the distance AD is very small, then the arc AD and the chord AD are practically equal. Draw the line DM at right angles to AE. The distance $DC = AM$ will then represent the amount that the point A is drawn out of the path AB.

The distance AD represents the space passed over by the point A in a short interval of time, say t sec., so that $AD = V \times t$. Now let a = the acceleration with which the point A is being drawn out of its path in a straight line. The distance AM will equal $\frac{1}{2} a \times t^2$ or $AM = \frac{1}{2} at^2$.

Draw the lines AE and DE. Then the triangles $ADMA$ and $AEDA$ will be similar, that is, they have their angles respectively equal, and as a result their sides will be proportional each to each. Thus,

$$\frac{AD}{AE} = \frac{AM}{AD}, \text{ or, } AD^2 = AM \times AE$$

From the above discussion, $AD = Vt$ and $AM = \frac{1}{2} at^2$, and as AE is the diameter of the circle, it must equal $2R$. Now, substituting these values in the above equation, there results

$$V^2t^2 = \tfrac{1}{2} at^2 \times 2 \times R,$$

or

$$V^2 = a \times R,$$

and

$$a = \frac{V^2}{R} \tag{78}$$

That is the acceleration with which the given point A is drawn out of its path in a straight line by the force acting in the link OA is $\frac{V^2}{R}$, and from equation (49) $F = \frac{W}{g}a$. Therefore, let C = the centripetal force exerted by the link OA to keep the point A moving at a distance R from the point O, then

$$C = \frac{W}{g} \times a = \frac{W \times V^2}{gR} \qquad (79)$$

This same equation may be used to figure the centrifugal force exerted by rotating bodies. The existence of this force accounts for the fact that all large rotating bodies, such as flywheels, must be carefully balanced, otherwise there will be an excessive pressure exerted on the bearings supporting the rotating body. Likewise, the crank pins and part of the connecting-rods of engines must be balanced. This may be done in various ways. Where crank disks are used, one end is made much heavier than the other. In locomotives, additional weight is added on the side of the driver opposite to that containing the crankpin. This is called the counterweight or counterbalance.

Equation (79) may be expressed directly in terms of the revolutions per minute, N, made by the wheel. Since the linear velocity $V = \frac{2\pi RN}{60}$ ft. per sec., it follows that

$$C = \frac{WV^2}{gR} = \frac{W \times (2\pi)^2 \times R^2 \times N^2}{60^2 \times g \times R} = \frac{(2\pi)^2 WRN^2}{3600 \times g}$$

or

$$C = 0.000341\ WRN^2 \qquad (80)$$

Study Questions

131. A flywheel 10 ft. in diameter is running at 100 r.p.m. Find the acceleration with which a point on the rim is moving toward the center of the wheel.

132. If the face of the wheel is 10 in. and the thickness of the rim 8 in., what is the centrifugal force per inch length of rim?

133. A wheel running at 1500 r.p.m. has an unbalanced weight of 5 lb. at a distance of 18 in. from the center. Find the pressure in pounds on the bearings due to this unbalanced weight.

134. Fig. 81 is a diagrammatic sketch of Fig. 14. If the link OA is 18 in. long, find the r.p.m. of the ball A when the link makes an angle of 30 deg. with the vertical.

135. What decrease in speed must take place so that the angle OAM will be reduced to 20 deg.?

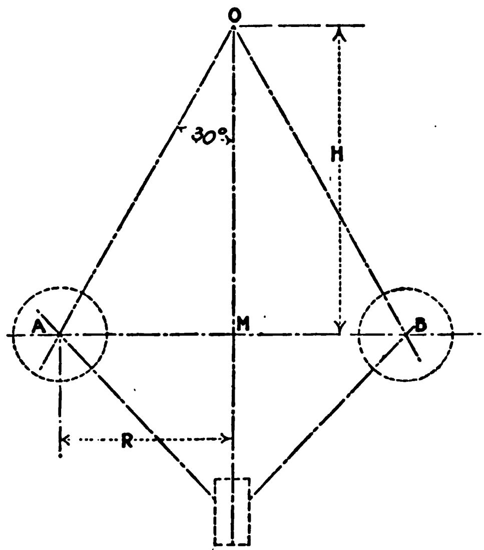

FIG. 81.

Answers

131. In equation (78) it was shown that the acceleration with which a point is drawn toward the center of a rotating body is $\frac{V^2}{R}$, where V is the linear velocity of the point, located R ft. from the center of rotation. In this case the linear velocity of a point on the rim of the wheel is $2 \times 3.1416 \times 5 \times 100 = 3141.6$ ft. per min. or 52.36 ft. per sec., since the radius of the rim is 5 ft. Therefore, the acceleration is

$$a = \frac{(52.36)^2}{5} = 548.3 \text{ ft. per sec. per sec.}$$

132. The weight of the rim may be found by the method explained in problem (128), thus

$$W = \frac{8 \times 10 \times 351.9}{1728} \times 450 = 7330 \text{ lb.}$$

The weight per inch length of rim is equal to the total weight of the rim divided by the circumference of a circle whose diameter is the mean diameter of the rim. Let w = the weight in pounds per inch length of rim, then $w = \left(\frac{7330}{351.9}\right) = 20.8$ lb., and the centrifugal force per inch length of rim is $\frac{wV^2}{gR}$, where V is the linear velocity of the point, located R ft. from the center of rotation, and equals

$$3.1416 \times (9\tfrac{1}{3}\text{ ft.}) \times \tfrac{100}{60} = 48.87 \text{ ft. per sec.}$$

Therefore the centrifugal force per inch of rim is

$$\frac{20.8 \times 48.87 \times 48.87}{32.16 \times (4\tfrac{2}{3}\text{ ft.})} = 331 \text{ lb.}$$

133. The pressure on the bearings is equal to the centrifugal force of the unbalanced weight of 5 lb. From equation (79) the centrifugal force is $\frac{WV^2}{gR}$, where W = 5 lb.; V = the velocity of the weight in feet per second $= \tfrac{1500}{60} \times 3.1416 \times 3 = 235.6$ ft. and R = the radius of gyration = 1.5 ft. Therefore, the centrifugal force is

$$C = \frac{5 \times 235.6 \times 235.6}{32.16 \times 1.5} = 5753 \text{ lb.}$$

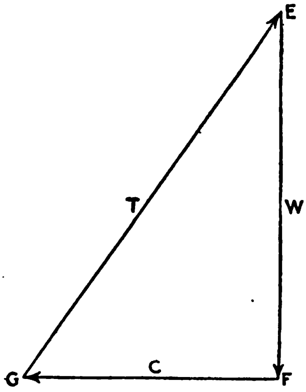

FIG. 82.

which gives the excess pressure on the bearings. This high speed was assumed to show the great strain that comes on the bearings supporting a shaft that holds an unbalanced wheel of any sort.

134. Assume only the weight of the balls A and B. In Fig. 15 it was shown that there are three forces acting on the ball A, and, as

these forces are in equilibrium, they may be represented by the sides of the triangle *EFGE*, Fig. 82, where the side *EF* is the weight of the ball; the side *GF* the centrifugal force *C*; and the side *GE* gives the tension in the supporting link. This force triangle has its sides respectively parallel to the triangle *OMAO*, Fig. 81, and therefore the two triangles are similar. Hence

$$\left(\frac{\text{centrifugal force } C}{\text{weight } W}\right) = \left(\frac{\text{side } AM}{\text{side } OM}\right) = \frac{R}{H}$$

and as $C = \frac{WV^2}{gR}$, it follows that

$$\frac{\left(\frac{WV^2}{gR}\right)}{W} = \frac{R}{H}, \text{ or } \frac{V^2}{g} = \frac{R^2}{H}$$

This equation shows that, *neglecting the friction*, the action of the flyball governor is independent of the weight of the ball. The velocity $V = 2 \times 3.1416 \times R \times N$ ft. per sec., where $N =$ the revolutions per second, hence,

$$\frac{(2 \times 3.1416 \times R \times N)^2}{32.16} = \frac{R^2}{H} \text{ or, } N^2 = \frac{0.815}{H}$$

But $H = OA \cos 30$ deg. $= 1.5$ ft. $\times 0.866 = 1.299$ ft. Therefore,

$$N^2 = \frac{0.815}{1.299} = 0.63 \text{ or } N = 0.79 \text{ rev. per sec.} = 47.4 \text{ r.p.m.}$$

135. If the angle *OAM* is 20 deg., then

$$H = 1.5 \times \cos 20 \text{ deg.} = 1.5 \times 0.94 = 1.41 \text{ ft.}$$

and

$$N^2 = \frac{0.815}{H} = \frac{0.815}{1.41} = 0.58,$$

or

$$N = 0.76 \text{ rev. per sec.} = 45.6 \text{ r.p.m.}$$

The reduction in speed $= (47.4 - 45.6) = 1.8$ r.p.m.

CHAPTER XIX

MECHANICS OF BELTING

One of the common questions raised in the power plant and in the shop is: What size belt shall be used to transmit a given horsepower? This may be answered by the application of the principles of mechanics outlined in the previous chapters. Thus the force of friction between two bodies is dependent upon the normal pressure and the coefficient of friction. In the case of a belt this force of friction tends to change the tension in the two sides of the belt.

In Fig. 83, if a belt be stretched over the two pulleys A and B (assumed idle) and spring balances be placed on the top and

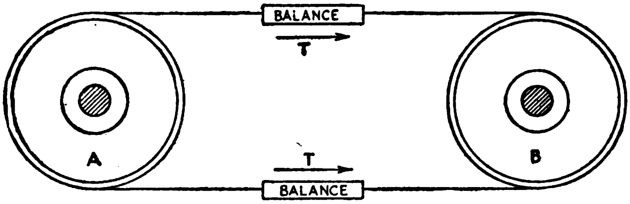

FIG. 83.

bottom sides of the belt it will be found that these balances will register approximately the same, tension T. Now if the pulley A be put in motion, it is a matter of experience that the lower or driving side of the belt will tend to lengthen and as a result the upper side will become slack. Stated in other words, the tension in the driving side increases and the tension in the slack side decreases. This difference in the tensions is due to the force of friction acting between the belt and the face of the pulley. In Fig. 84 let the pulley A be the driver and the pulley B the follower. Let T_1 be the tension in the tight or

driving side of the belt and T_2 be the tension in the slack side. Let a = the arc of contact between the belt and the face of the smaller pulley B, expressed in radians; and let f = the coefficient of friction of the belt. By the aid of higher mathematics, it may be shown that the relation between the tensions of the tight and slack sides of the belt is expressed by the equation

$$\log_e\left(\frac{T_1}{T_2}\right) = f \times a \tag{81}$$

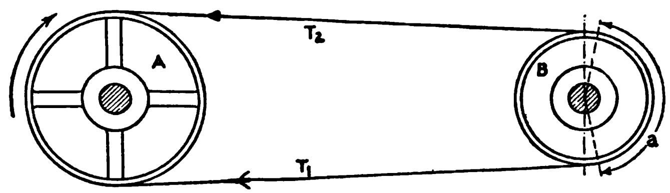

FIG. 84.

where e is the Naperian base of logarithms. This equation may be expressed in simpler form by changing the Naperian to the common logarithm from the fact that the common log = 0.434 × the Naperian log, hence,

$$\log_{10}\left(\frac{T_1}{T_2}\right) = 0.434 f \times a \tag{82}$$

which may still further be simplified by inserting in place of a radians its equivalent value in degrees. Let N = the ratio of the arc of contact to the circumference of the smaller pulley. If there are a radians in the arc of contact, then $N = \frac{a}{2\pi}$ or $a = 2\pi N$ (since there are 2π radians in the circumference of a circle). This ratio N might also be found by dividing the arc of contact in degrees by 360 (the number of degrees in the circumference of a circle). Now substituting the value of $a = 2\pi N$ in equation (82), there results:

$$\log_{10}\left(\frac{T_1}{T_2}\right) = 0.434 \times f \times 2\pi \times N = 2.73\, fN \tag{83}$$

If the distance between the pulleys is large and the diameters are nearly the same, then N will equal approximately 0.5. Under average conditions, the coefficient of friction f will equal about 0.3 so that

$$\log_{10}\frac{T_1}{T_2} = 2.73 \times 0.3 \times 0.5 = 0.409$$

and from a table, the value of log $\left(\frac{T_1}{T_2}\right)$ is found to be approximately 2.5. The effective driving effort P, exerted by the belt will be the difference between the tensions in the tight and slack sides or,

$$P = T_1 - T_2 \tag{84}$$

If the velocity of the belt is V ft. per min., then the work done in foot-pounds per minute will equal $P \times V$ (force times distance) and the horsepower transmitted by the belt will be given by the equation,

$$\text{hp.} = \frac{P \times V}{33{,}000} \tag{85}$$

Example.—A single leather belt $\frac{5}{32}$ in. thick is capable of standing about 400 lb. per sq. in., so that for every inch of width a working tension of $400 \times \frac{5}{32}$, or 62 lb., may be placed on the belt. Derive a simple rule for the horsepower transmitted by a single belt.

Solution.—Assuming an arc of contact of 180 deg. and a coefficient of friction = 0.3, the relation between T_1 and T_2 will be as previously found, namely, $\frac{T_1}{T_2} = 2.5$, or $T_2 = 0.4\ T_1$. The effective driving effort

$$P = (T_1 - T_2) = (T_1 - 0.4\ T_1) = 0.6\ T_1$$

Now the allowable pull is 62 lb. per in. of width so that the effective pull is

$$P = 0.6\ T_1 = 0.6 \times 62 = 37.2 \text{ lb. per in. width of belt.}$$

One horsepower = 33,000 ft.-lb. The work done by the belt equals PV ft.-lb. per min. so that the velocity required of a single belt 1 in. wide to deliver 1 hp. must equal $\frac{33,000}{37.2}$ or 885 ft. per min.

The above problem is the basis for the commonly accepted rule that a *single leather belt traveling at* 900 *ft. per min. will transmit* 1 *hp. for every inch of its width.* Theoretically, a double belt will transmit twice the horsepower of a single belt but actually it will only transmit about 1.4 times as much.

Study Questions

136. The distance between the centers of two pulleys, 18 in. and 30 in. in diameter, is 10 ft. Find the arc of contact on the smaller pulley, assuming an open belt.

137. If the coefficient of friction is 0.3, find the relation between T_1 and T_2 in the above problem.

138. If the smaller pulley makes 300 r.p.m., find the horsepower transmitted by a belt 5/32 in. thick and 12 in. wide, assuming that the belt is good for 60 lb. per in. of width.

139. A belt is traveling at the rate of 2700 ft. per min. Find by the approximate rule the horsepower transmitted by a single belt 15 in. wide.

140. If a double belt were used, what would be the horsepower transmitted?

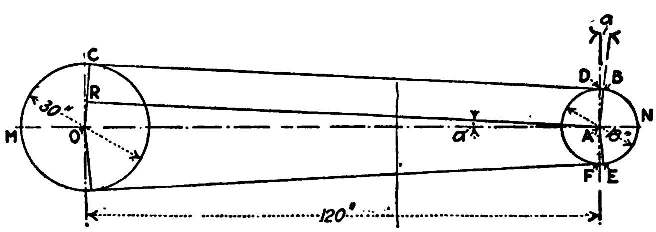

FIG. 85.

Answers

136. In Fig. 85 let the distance between the centers of the pulleys M and N be 10 ft. or 120 in. Draw the line of the belt. Its arc of contact on the small pulley will be the angle BAE which is equal to 180 deg. minus twice the angle DAB. From the center of the small pulley N

draw the line AR parallel to the line CB. Then since the line AR is at right angles to the line AB and the line AO makes 90 deg. with the line AD it is evident that angle DAB = angle a = angle RAO. The line OR is equal to the radius of the large pulley minus the radius of the small pulley or line $OR = (15 - 9) = 6$ in. The sine of the angle RAO is equal to $\frac{OR}{OA} = 6/120 = 0.05$ and from a table of sines the value of a is found to be (2 deg. 50 min.). Therefore, the arc of contact is

$$(180 - 2 \times 2 \text{ deg. } 50 \text{ min.}) = 174 \text{ deg. } 20 \text{ min.}$$

137. From equation (83) $\log_{10}\left(\frac{T_1}{T_2}\right) = 2.73 \times f \times N$, where f = the coefficient of friction = 0.3, and N = the ratio of the arc of contact in degrees to 360 deg. or $N = \frac{174 \text{ deg. } 20 \text{ min.}}{360} = 0.484$. Hence

$$\log_{10}\left(\frac{T_1}{T_2}\right) = 2.73 \times 0.3 \times 0.484 = 0.396$$

and from a table of logarithms the value of $\frac{T_1}{T_2}$ is found to be 2.49 or $T_1 = 2.49\ T_2$.

138. Since the belt is 12 in. wide and good for 60 lb. per in. of width the maximum tension T_1 that may be put on the belt = (12 × 60) or 720 lb. The effective driving effort $P = (T_1 - T_2)$ and from the above problem $T_1 = 2.49\ T_2$, or $T_2 = 0.4\ T_1$; hence

$$P = (T_1 - 0.4\ T_1) = 0.6\ T_1 = (0.6 \times 720) = 432 \text{ lb.}$$

The linear velocity of the belt = r.p.m. × the circumference of the smaller pulley or

$$V = 300 \times 3.1416 \times 1.5 = 1413 \text{ ft. per min.}$$

and from equation (55)

$$\text{hp.} = \frac{P \times V}{33{,}000} = \frac{432 \times 1413}{33{,}000} = 18.5$$

139. The rule states that a belt traveling at 900 ft. per min. will transmit 1 h.p. per in. width of belt. If traveling at 2700 ft. the belt will evidently transmit $\frac{2700}{900}$ or 3 hp. per in. of width and as the total width of the belt is 15 in. the total horsepower transmitted will equal 15 × 3 or 45 hp.

140. The horsepower transmitted by a double belt = 1.4 times that of a single belt, hence in this case a double belt would transmit 1.4 × 45 or 63 hp.

CHAPTER XX

REVIEW

Mechanics is that branch of science which has to do with forces and their action on bodies tending to produce a state of motion, change of motion, or condition of rest of these bodies. This study of forces is divided into two general classes of statics and kinetics, the former dealing only with forces which keep a body in a state of rest and the latter dealing with forces which produce motion. The first nine lessons (Chapter I to VI) of this course covered the general idea of statics, and Lesson X (Chapter VII) gave a résumé of the conditions which produce a state of equilibrium of all the forces acting so as to insure a state of rest of the bodies.

In Lessons XII, XIII and XIV (Chapter VIII) these laws of equilibrium were applied to the various forms of machines such as levers and pulleys including the Weston differential pulley. This idea of a machine naturally led to Lesson XIV (Chapter IX), where a few general notions of friction were given. It was shown that the force of friction acting between two bodies was directly proportional to the coefficient of friction times the normal pressure between the objects. To find the normal pressure it is necessary to resolve all the forces acting on the body into their components at right angles to the bodies. The algebraic sum of all these components will give the total pressure tending to produce friction. The algebraic sum of the components parallel to the bodies will produce or tend to produce motion of the bodies one on the other.

With the force of friction overcome, the next logical step is to study the laws which produce motion of bodies and to determine under what conditions a system of forces will be

in equilibrium even *though the objects are in motion at a uniform rate of speed.* In Lesson XV (Chapter X) the general conception of motion was dwelt upon and as in statics so in kinetics it was found that velocities might be represented by straight lines; that they might be reduced into components in any desired direction, and moreover that a system of velocities could be replaced by a single velocity which would produce exactly the same motion as the combined action of all the other velocities.

The relations between the acceleration, velocity, time and space traversed by bodies were worked out in considerable detail and for handy reference these formulas[1] may be summarized as follows, first, however, dividing moving bodies into six general classes, thus:

Class I. Bodies having uniform motion.

Class II. Bodies starting from rest and moving with uniformly accelerated motion.

Class III. Bodies moving with an initial velocity and then acted upon by an accelerating or retarding force.

Class IV. Bodies falling freely under the action of gravity.

Class V. Projectiles, subdivided into:
(a) Bodies projected horizontally from a point at some elevation above a given reference plane.
(b) Bodies projected at an angle to the horizontal.

Class IV. Rotating bodies.

All problems coming under Class I may be solved by the relation that,

$$S = V \times T \tag{27}$$

where V = The uniform linear velocity in feet per second.
S = The space traversed in feet.
T = The total time of motion in seconds.

[1] The numbers opposite the equations are the same as used in the various lessons.

Problems coming under Class II are solved by the following laws:

$$V = a \times T \tag{30}$$
$$S = \tfrac{1}{2} a \times T^2 \tag{31}$$
$$V^2 = 2a \times S \tag{32}$$

where V = The final velocity in feet per second.
S = The space traversed in feet.
T = The total time in seconds.
a = The acceleration in feet per second per second.

The formulas for problems coming under Class III are:

$$S = U \times T \pm \tfrac{1}{2} a \times T^2 \qquad (34) \text{ and } (38)$$
$$V_1 = U \pm a \times T \qquad (35) \text{ and } (39)$$
$$V_1^2 = U^2 \pm 2a \times S \qquad (37) \text{ and } (40)$$

where
U = The initial velocity in feet per second.
V_1 = The final velocity in feet per second.
S = The space traversed in feet.
a = The acceleration or deceleration in feet per second per second.

The relation between the various items for falling bodies under Class IV are:

$$V = g \times T_1 \tag{41}$$
$$H = \tfrac{1}{2} g \times T_1^2 \tag{42}$$
$$V^2 = 2g \times H \tag{43}$$

where H = The height of fall in feet.
V = The velocity at the end of a fall of T_1 sec. duration.
g = 32.16 = the acceleration in feet per second per second due to gravity.

The law for projectiles under Class V, Case a, are expressed by the relations:

$$T_1 = \sqrt{\frac{2H}{g}} \tag{67}$$

$$R = V \times T_1 \tag{68}$$

$$R = V\sqrt{\frac{2H}{g}} \tag{69}$$

and for Class V, Case *b*:

$$T_1 = \frac{2V \times \sin a}{g} \tag{70}$$

$$R = V \cos a \times T_1 \tag{71}$$

$$R = \frac{V^2 \sin 2a}{g} \tag{72}$$

$$H = \frac{V^2 \sin^2 a}{2g} \tag{73}$$

where T_1 = The total time of flight in seconds.
H = The maximum altitude reached by the projectile or is equal to the elevation of the point of projection in feet.
R = The range in feet.
V = The initial velocity of projection in feet per second.
a = The angle of projection in degrees.
g = 32.16 = the acceleration in feet per second per second due to gravity.

The most common law for rotating bodies as described under Class VI may be stated as an equation which gives the centrifugal force of a weight of W lb., rotating at N revolutions per minute at a distance of R ft. from a fixed point of rotation, thus,

$$C = 0.000341\, W \times R \times N^2 \tag{80}$$

With the idea of motion clearly fixed in mind, it is an easy step to the conception of the term work which has been defined as the overcoming of a resistance through a given distance. The formulas for the work done by various types of machines such as engines, pumps, motors, etc., are almost innumerable. If the student will very carefully analyze his problem he will

find that in all probability all his examples relating to work may be solved by the general equation that,

$$\text{horsepower} = \frac{\text{force in pounds} \times \text{distance in feet}}{\text{time in minutes} \times \text{efficiency} \times 33{,}000}$$

Indicated horsepower, brake horsepower, output and input of pumps, generators, etc., may all be found by applying the above equation with the use of the proper *coefficient of common sense, which is, after all, the most essential factor in the solution of any and all problems no matter how simple or how difficult.*

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